3-1 Lines and Angles

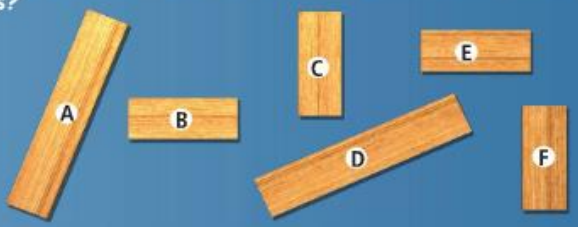
Objectives To identify relationships between figures in space
To identify angles formed by two lines and a transversal

Try visualizing how the bookcase looks in two dimensions.

SOLVE IT!

Getting Ready!

You want to assemble a bookcase. You have all the pieces, but you misplaced the instructions that came with the box. How would you write the instructions?



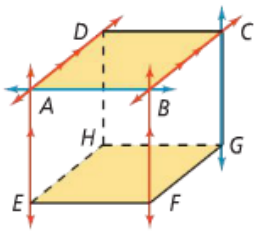
Lesson Vocabulary
- parallel lines
- skew lines
- parallel planes
- transversal
- alternate interior angles
- same-side interior angles
- corresponding angles
- alternate exterior angles

In the Solve It, you used relationships among planes in space to write the instructions. In Chapter 1, you learned about intersecting lines and planes. In this lesson, you will explore relationships of nonintersecting lines and planes.

Essential Understanding Not all lines and not all planes intersect.

take note

Key Concept Parallel and Skew

Definition	Symbols	Diagram
Parallel lines are coplanar lines that do not intersect. The symbol $\parallel$ means "is parallel to."	$\overleftrightarrow{AE} \parallel \overleftrightarrow{BF}$ $\overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$	
Skew lines are noncoplanar; they are not parallel and do not intersect.	$\overleftrightarrow{AB}$ and $\overleftrightarrow{CG}$ are skew.	
Parallel planes are planes that do not intersect.	plane $ABCD \parallel$ plane $EFGH$	

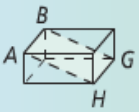
Use arrows to show $\overleftrightarrow{AE} \parallel \overleftrightarrow{BF}$ and $\overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$.

A line and a plane that do not intersect are parallel. Segments and rays can also be parallel or skew. They are parallel if they lie in parallel lines and skew if they lie in skew lines.

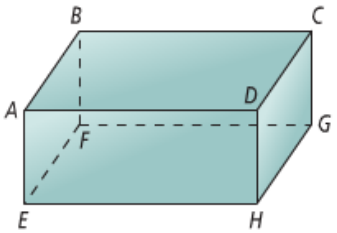
Problem 1 Identifying Nonintersecting Lines and Planes

Think

Parallel lines are coplanar. Which planes contain $\overline{AB}$?
Planes *ABCD*, *ABFE*, and *ABGH* contain $\overline{AB}$. You need to visualize plane *ABGH*.



In the figure, assume that lines and planes that appear to be parallel are parallel.



A Which segments are parallel to $\overline{AB}$?

$\overline{EF}$, $\overline{DC}$, and $\overline{HG}$

B Which segments are skew to $\overline{CD}$?

$\overline{BF}$, $\overline{AE}$, $\overline{EH}$, and $\overline{FG}$

C What are two pairs of parallel planes?

plane *ABCD* ∥ plane *EFGH*

plane *DCG* ∥ plane *ABF*

D What are two segments parallel to plane *BCGF*?

$\overline{AD}$ and $\overline{DH}$

Got It? **1.** Use the figure in Problem 1.

a. Which segments are parallel to $\overline{AD}$?

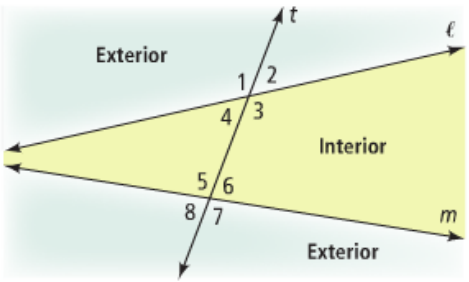
b. Reasoning Explain why $\overline{FE}$ and $\overline{CD}$ are *not* skew.

c. What is another pair of parallel planes?

d. What are two segments parallel to plane *DCGH*?

Essential Understanding When a line intersects two or more lines, the angles formed at the intersection points create special angle pairs.

A **transversal** is a line that intersects two or more coplanar lines at distinct points. The diagram below shows the eight angles formed by a transversal t and two lines ℓ and m.



Notice that angles 3, 4, 5, and 6 lie between ℓ and m. They are *interior* angles. Angles 1, 2, 7, and 8 lie outside of ℓ and m. They are *exterior* angles.

Pairs of the eight angles have special names as suggested by their positions.

take note

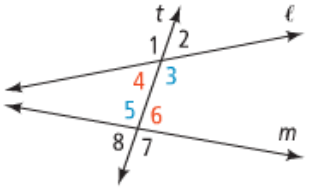
Key Concept Angle Pairs Formed by Transversals

Definition

Alternate interior angles are nonadjacent interior angles that lie on opposite sides of the transversal.

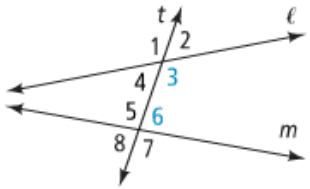
Example

∠4 and ∠6
∠3 and ∠5



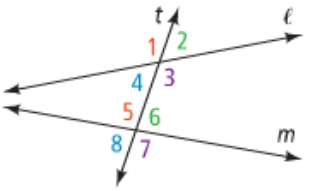
Same-side interior angles are interior angles that lie on the same side of the transversal.

∠4 and ∠5
∠3 and ∠6



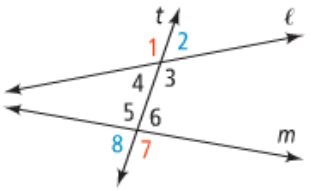
Corresponding angles lie on the same side of the transversal *t* and in corresponding positions.

∠1 and ∠5
∠4 and ∠8
∠2 and ∠6
∠3 and ∠7



Alternate exterior angles are nonadjacent exterior angles that lie on opposite sides of the transversal.

∠1 and ∠7
∠2 and ∠8

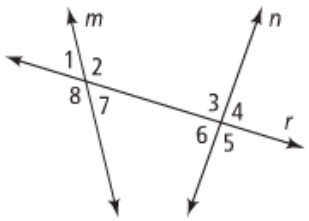


ONLINE PROBLEMS

Problem 2 Identifying an Angle Pair

Multiple Choice Which is a pair of alternate interior angles?

(A) ∠1 and ∠3
(B) ∠6 and ∠7
(C) ∠2 and ∠6
(D) ∠4 and ∠8



∠2 and ∠6 are alternate interior angles because they lie on opposite sides of the transversal *r* and in between *m* and *n*. The correct answer is C.

Think

Which choices can you eliminate?
You need a pair of interior angles. ∠1, ∠4, and ∠8 are exterior angles. You can eliminate choices A and D.

Got It? **2.** Use the figure in Problem 2. What are three pairs of corresponding angles?

Problem 3 Classifying an Angle Pair

Architecture The photo below shows the Royal Ontario Museum in Toronto, Canada. Are angles 2 and 4 *alternate interior angles, same-side interior angles, corresponding angles,* or *alternate exterior angles*?

Think

How do the positions of ∠2 and ∠4 compare?
∠2 and ∠4 are both interior angles and they lie on opposite sides of a line.

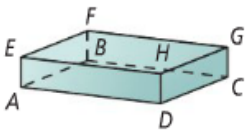
Angles 2 and 4 are alternate interior angles.

Got It? **3.** In Problem 3, are angles 1 and 3 *alternate interior angles, same-side interior angles, corresponding angles,* or *alternate exterior angles*?

Lesson Check

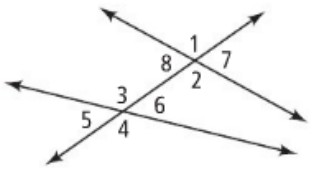
Do you know HOW?

Name one pair each of the segments, planes, or angles. Lines and planes that appear to be parallel are parallel.



Exercises 1–3

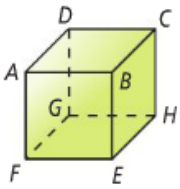
1. parallel segments
2. skew segments
3. parallel planes
4. alternate interior
5. same-side interior
6. corresponding
7. alternate exterior



Exercises 4–7

Do you UNDERSTAND?

8. **Vocabulary** Why is the word *coplanar* included in the definition for parallel lines?
9. **Vocabulary** How does the phrase *alternate interior angles* describe the positions of the two angles?
10. **Error Analysis** In the figure at the right, lines and planes that appear to be parallel are parallel. Carly says $\overline{AB} \parallel \overline{HG}$. Juan says $\overline{AB}$ and $\overline{HG}$ are skew. Who is correct? Explain.



Practice and Problem-Solving Exercises

A Practice

Use the diagram to name each of the following. Assume that lines and planes that appear to be parallel are parallel. See Problem 1.

11. a pair of parallel planes

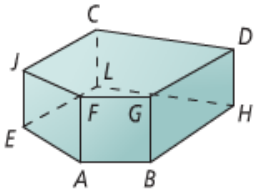
12. all lines that are parallel to $\overleftrightarrow{AB}$

13. all lines that are parallel to $\overleftrightarrow{DH}$

14. two lines that are skew to $\overleftrightarrow{EJ}$

15. all lines that are parallel to plane *JFAE*

16. a plane parallel to $\overleftrightarrow{LH}$



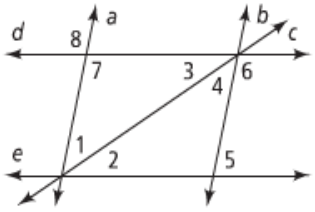
Identify all pairs of each type of angles in the diagram. Name the two lines and the transversal that form each pair. See Problem 2.

17. corresponding angles

18. alternate interior angles

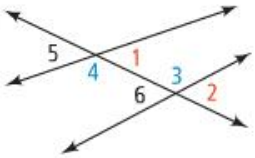
19. same-side interior angles

20. alternate exterior angles

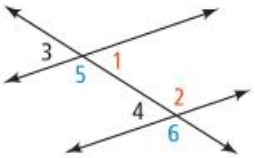


Are the angles labeled in the same color *alternate interior angles, same-side interior angles, corresponding angles,* or *alternate exterior angles*? See Problem 3.

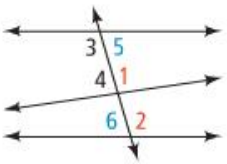
21.



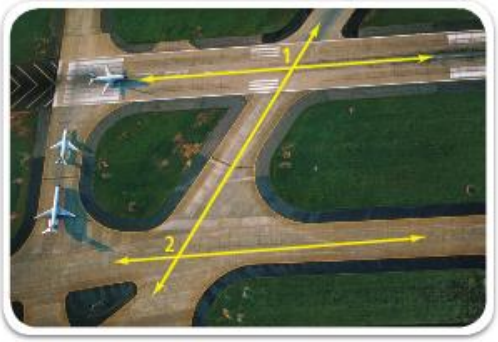
22.



23.



24. Aviation The photo shows an overhead view of airport runways. Are $\angle 1$ and $\angle 2$ *alternate interior angles, same-side interior angles, corresponding angles,* or *alternate exterior angles*?



B Apply

How many pairs of each type of angles do two lines and a transversal form?

25. alternate interior angles

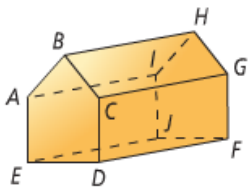
26. corresponding angles

27. alternate exterior angles

28. vertical angles

29. Recreation You and a friend are driving go-karts on two different tracks. As you drive on a straight section heading east, your friend passes above you on a straight section heading south. Are these sections of the two tracks *parallel, skew,* or *neither*? Explain.

In Exercises 30–35, describe the statement as *true* or *false*. If false, explain. Assume that lines and planes that appear to be parallel are parallel.



30. $\overleftrightarrow{CB} \parallel \overleftrightarrow{HG}$

31. $\overleftrightarrow{ED} \parallel \overleftrightarrow{HG}$

32. plane $AED \parallel$ plane FGH

33. plane $ABH \parallel$ plane CDF

34. $\overleftrightarrow{AB}$ and $\overleftrightarrow{HG}$ are skew lines.

35. $\overleftrightarrow{AE}$ and $\overleftrightarrow{BC}$ are skew lines.

36. Think About a Plan A rectangular rug covers the floor in a living room. One of the walls in the same living room is painted blue. Are the rug and the blue wall parallel? Explain.

- Can you visualize the rug and the wall as geometric figures?
- What must be true for these geometric figures to be parallel?

In Exercises 37–42, determine whether each statement is *always, sometimes,* or *never* true.

37. Two parallel lines are coplanar.

38. Two skew lines are coplanar.

39. Two planes that do not intersect are parallel.

40. Two lines that lie in parallel planes are parallel.

41. Two lines in intersecting planes are skew.

42. A line and a plane that do not intersect are skew.

43. a. Writing Describe the three ways in which two lines may be related.

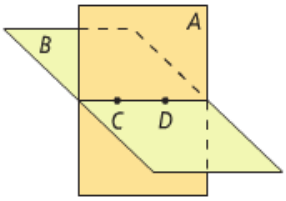
b. Give examples from the real world to illustrate each of the relationships you described in part (a).

44. Open-Ended The letter Z illustrates alternate interior angles. Find at least two other letters that illustrate pairs of angles presented in this lesson. Draw the letters. Then mark and describe the angles.

45. a. Reasoning Suppose two parallel planes A and B are each intersected by a third plane C. Make a conjecture about the intersection of planes A and C and the intersection of planes B and C.

b. Find examples in your classroom to illustrate your conjecture in part (a).

Challenge **Use the figure at the right for Exercises 46 and 47.**



46. Do planes A and B have other lines in common that are parallel to $\overleftrightarrow{CD}$? Explain.

47. Visualization Are there planes that intersect planes A and B in lines parallel to $\overleftrightarrow{CD}$? Draw a sketch to support your answer.

48. Draw a Diagram A transversal r intersects lines ℓ and m. If ℓ and r form $\angle 1$ and $\angle 2$ and m and r form $\angle 3$ and $\angle 4$, sketch a diagram that meets the following conditions.

- $\angle 1 \cong \angle 2$
- $\angle 3$ is an interior angle.
- $\angle 4$ is an exterior angle.
- $\angle 3$ and $\angle 4$ are supplementary.
- $\angle 2$ and $\angle 4$ lie on opposite sides of r.

Standardized Test Prep

SAT/ACT

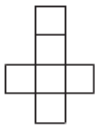
49. How many pairs of parallel planes does a cereal box have?

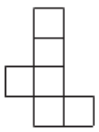
(A) 2 (B) 3 (C) 4 (D) 6

50. What are the coordinates of the midpoint of $\overline{AB}$ for $A(-2, 8)$ and $B(-4, 4)$?

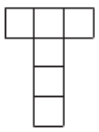
(F) $(-6, 12)$ (G) $(-3, 6)$ (H) $(1, 2)$ (I) $(1, 6)$

51. Which of the following is NOT the net of a cube?

(A) 

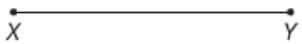
(B) 

(C)

(D) 

Short Response

52. Construct $\overline{MN}$ congruent to $\overline{XY}$.



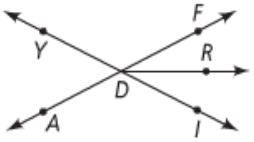
Mixed Review

If $m\angle YDF = 121$ and $\overrightarrow{DR}$ bisects $\angle FDI$, find the measure of each angle. See Lesson 2-6.

53. $\angle IDA$

54. $\angle YDA$

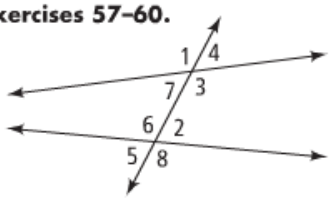
55. $\angle RDI$



56. What are the next two terms in the sequence $1, -2, 4, -8, \ldots$? See Lesson 2-1.

Get Ready! **To prepare for Lesson 3-2, do Exercises 57–60.**

Classify each pair of angles. See Lesson 3-1.



57. $\angle 4$ and $\angle 2$

58. $\angle 6$ and $\angle 3$

59. $\angle 4$ and $\angle 5$

60. $\angle 6$ and $\angle 7$

Concept Byte

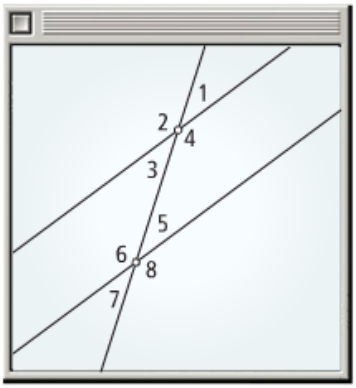
Use With Lesson 3-2

TECHNOLOGY

Parallel Lines and Related Angles

Activity

Use geometry software to construct two parallel lines. Check that the lines remain parallel as you manipulate them. Construct a point on each line. Then construct the transversal through these two points.



1. Measure each of the eight angles formed by the parallel lines and the transversal. Record the measurements.
2. Manipulate the lines. Record the new measurements.
3. When a transversal intersects parallel lines, what are the relationships among the angle pairs formed? Make as many conjectures as possible.

Exercises

4. Construct three or more parallel lines. Then construct a line that intersects all the parallel lines.
   a. What relationships can you find among the angles formed?
   b. How many different angle measures are there?
5. Construct two parallel lines and a transversal perpendicular to one of the parallel lines. What angle does the transversal form with the second line?
6. Construct two lines and a transversal, making sure that the two lines are *not* parallel. Locate a pair of alternate interior angles. Manipulate the lines so that these angles have the same measure.
   a. Make a conjecture about the relationship between the two lines.
   b. How is this conjecture different from the conjecture(s) you made in the Activity?
7. Again, construct two lines and a transversal, making sure that the two lines are *not* parallel. Locate a pair of same-side interior angles. Manipulate the lines so that these angles are supplementary.
   a. Make a conjecture about the relationship between the two lines.
   b. How is this conjecture different from the conjecture(s) you made in the Activity?
8. Construct perpendicular lines a and b. At a point that is not the intersection of a and b, construct line c perpendicular to line a. Make a conjecture about lines b and c.

3-2 Properties of Parallel Lines

Objectives To prove theorems about parallel lines
To use properties of parallel lines to find angle measures

You see the vertical angles, right? Keep looking! There are other angle pairs.

SOLVE IT!

Getting Ready!

Look at the map of streets in Clearwater, Florida. Nicholson Street and Cedar Street are parallel. Which pairs of angles appear to be congruent?

In the Solve It, you identified several pairs of angles that appear congruent. You already know the relationship between vertical angles. In this lesson, you will explore the relationships between the angles you learned about in Lesson 3-1 when they are formed by *parallel* lines and a transversal.

Essential Understanding The special angle pairs formed by parallel lines and a transversal are congruent, supplementary, or both.

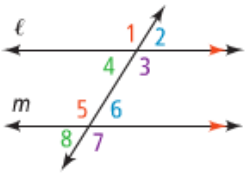
take note

Postulate 3-1 Corresponding Angles Postulate

Postulate
If a transversal intersects two parallel lines, then corresponding angles are congruent.

If . . .

$\ell \parallel m$



Then . . .

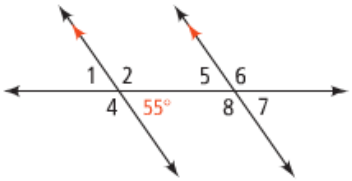
$\angle 1 \cong \angle 5$
$\angle 2 \cong \angle 6$
$\angle 3 \cong \angle 7$
$\angle 4 \cong \angle 8$

Problem 1 Identifying Congruent Angles

Think

How do you find angles congruent to the given 55° angle?
Look for angles that are corresponding and vertical to the given angle.

Which angles measure 55? How do you know?



$m\angle 7 = 55$ by the Corresponding Angles Postulate.

$m\angle 1 = 55$ by the Vertical Angles Theorem.

$m\angle 5 = 55$ by the Corresponding Angles Postulate because $\angle 1$ and $\angle 5$ are corresponding angles.

Got It? **1. a. Reasoning** In Problem 1, can you find another way to justify $m\angle 5 = 55$? Explain.

b. Using linear pairs, find $m\angle 4$. Which other angles have that measure? How do you know?

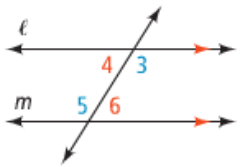
You can use the Corresponding Angles Postulate to prove other angle relationships.

take note

Theorem 3-1 Alternate Interior Angles Theorem

Theorem
If a transversal intersects two parallel lines, then alternate interior angles are congruent.

If . . .
$\ell \parallel m$

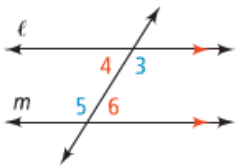


Then . . .
$\angle 4 \cong \angle 6$
$\angle 3 \cong \angle 5$

Theorem 3-2 Same-Side Interior Angles Theorem

Theorem
If a transversal intersects two parallel lines, then same-side interior angles are supplementary.

If . . .
$\ell \parallel m$



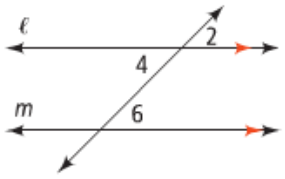
Then . . .
$m\angle 4 + m\angle 5 = 180$
$m\angle 3 + m\angle 6 = 180$

You will prove Theorem 3-2 in Exercise 25.

Proof Proof of Theorem 3-1: Alternate Interior Angles Theorem

Given: $\ell \parallel m$

Prove: $\angle 4 \cong \angle 6$

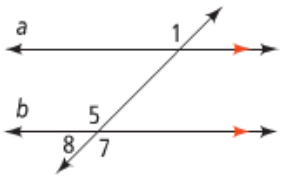


Statement	Reasons
1) $\ell \parallel m$	1) Given
2) $\angle 2 \cong \angle 6$	2) If lines are $\parallel$, then corresponding $\measuredangle$s are $\cong$.
3) $\angle 4 \cong \angle 2$	3) Vertical $\measuredangle$s are $\cong$.
4) $\angle 4 \cong \angle 6$	4) Transitive Prop. of $\cong$

Proof ONLINE PROBLEM VIDEO **Problem 2** **Proving an Angle Relationship**

Given: $a \parallel b$

Prove: $\angle 1$ and $\angle 8$ are supplementary.



Know

- $a \parallel b$

From the diagram you know

- $\angle 1$ and $\angle 5$ are corresponding
- $\angle 5$ and $\angle 8$ form a linear pair

Need

$\angle 1$ and $\angle 8$ are supplementary, or $m\angle 1 + m\angle 8 = 180$.

Plan

Show that $\angle 1 \cong \angle 5$ and that $m\angle 5 + m\angle 8 = 180$. Then substitute $m\angle 1$ for $m\angle 5$ to prove that $\angle 1$ and $\angle 8$ are supplementary.

Statements	Reasons
1) $a \parallel b$	1) Given
2) $\angle 1 \cong \angle 5$	2) If lines are $\parallel$, then corresp. $\measuredangle$s are $\cong$.
3) $m\angle 1 = m\angle 5$	3) Congruent $\measuredangle$s have equal measures.
4) $\angle 5$ and $\angle 8$ are supplementary.	4) $\measuredangle$s that form a linear pair are suppl.
5) $m\angle 5 + m\angle 8 = 180$	5) Def. of suppl. $\measuredangle$s
6) $m\angle 1 + m\angle 8 = 180$	6) Substitution Property
7) $\angle 1$ and $\angle 8$ are supplementary.	7) Def. of suppl. $\measuredangle$s

Got It? **2.** Using the same given information and diagram in Problem 2, prove that $\angle 1 \cong \angle 7$.

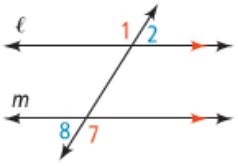
In the diagram for Problem 2, $\angle 1$ and $\angle 7$ are alternate exterior angles. In Got It 2, you proved the following theorem.

take note

Theorem 3-3 Alternate Exterior Angles Theorem

Theorem
If a transversal intersects two parallel lines, then alternate exterior angles are congruent.

If . . .
$\ell \parallel m$

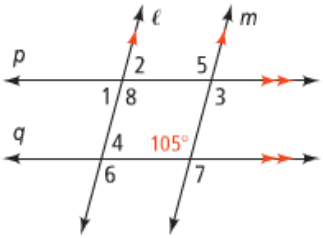


Then . . .
$\angle 1 \cong \angle 7$
$\angle 2 \cong \angle 8$

If you know the measure of one of the angles formed by two parallel lines and a transversal, you can use theorems and postulates to find the measures of the other angles.

Problem 3 Finding Measures of Angles

What are the measures of $\angle 3$ and $\angle 4$? Which theorem or postulate justifies each answer?



Think

How do $\angle 3$ and $\angle 4$ relate to the given 105° angle?
$\angle 3$ and the given angle are alternate interior angles. $\angle 4$ and the given angle are same-side interior angles.

Since $p \parallel q$, $m\angle 3 = 105$ by the Alternate Interior Angles Theorem.

Since $\ell \parallel m$, $m\angle 4 + 105 = 180$ by the Same-Side Interior Angles Theorem.
So, $m\angle 4 = 180 - 105 = 75$.

Got It? **3.** Use the diagram in Problem 3. What is the measure of each angle? Justify each answer.

a. $\angle 1$
b. $\angle 2$
c. $\angle 5$
d. $\angle 6$
e. $\angle 7$
f. $\angle 8$

You can combine theorems and postulates with your knowledge of algebra to find angle measures.

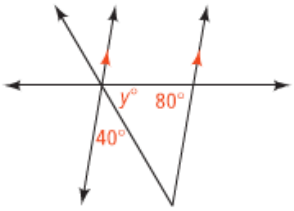
Problem 4 Finding an Angle Measure

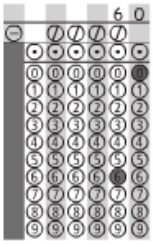
GRIDDED RESPONSE

Algebra What is the value of y?

Think

What do you know from the diagram?
You have one pair of parallel lines. The 80° angle and the angle formed by the 40° and $y°$ angles are same-side interior angles.





By the Angle Addition Postulate, $y + 40$ is the measure of an interior angle.

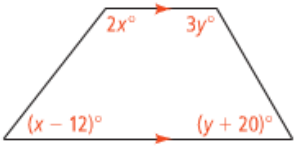
$(y + 40) + 80 = 180$ Same-side interior ∡s of ∥ lines are suppl.

$y + 120 = 180$ Simplify.

$y = 60$ Subtract 120 from each side.

Got It? **4. a.** In the figure at the right, what are the values of x and y?

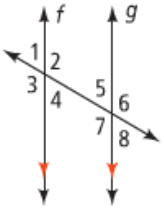
b. What are the measures of the four angles in the figure?



Lesson Check

Do you know HOW?

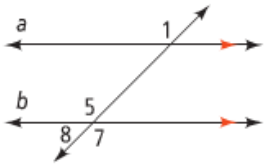
Use the diagram for Exercises 1–4.



1. Identify four pairs of congruent angles. (Exclude vertical angle pairs.)
2. Identify two pairs of supplementary angles. (Exclude linear pairs.)
3. If $m\angle 1 = 70$, what is $m\angle 8$?
4. If $m\angle 4 = 70$ and $m\angle 7 = 2x$, what is the value of x?

Do you UNDERSTAND?

5. **Compare and Contrast** How are the Alternate Interior Angles Theorem and the Alternate Exterior Angles Theorem alike? How are they different?
6. In Problem 2, you proved that $\angle 1$ and $\angle 8$, in the diagram below, are supplementary. What is a good name for this pair of angles? Explain.

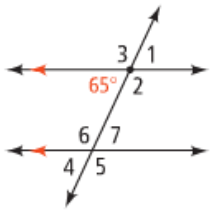


Practice and Problem-Solving Exercises

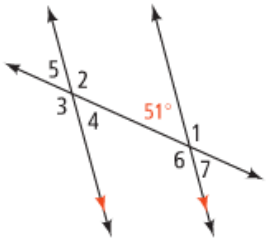
A Practice

Identify all the numbered angles that are congruent to the given angle. Justify your answers. See Problem 1.

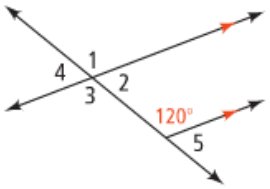
7.



8.



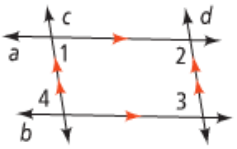
9.



10. Developing Proof Supply the missing reasons in the two-column proof. See Problem 2.

Given: $a \parallel b, c \parallel d$

Prove: $\angle 1 \cong \angle 3$



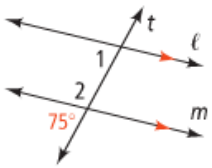
Statements	Reasons
1) $a \parallel b$	1) Given
2) $\angle 3$ and $\angle 2$ are supplementary.	2) a. ?
3) $c \parallel d$	3) Given
4) $\angle 1$ and $\angle 2$ are supplementary.	4) b. ?
5) $\angle 1 \cong \angle 3$	5) c. ?

11. Write a two-column proof for Exercise 10 that does not use $\angle 2$.

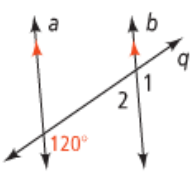
Proof

Find $m\angle 1$ and $m\angle 2$. Justify each answer. See Problem 3.

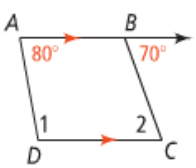
12.



13.

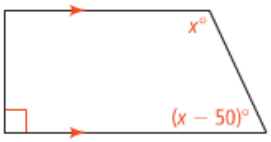


14.

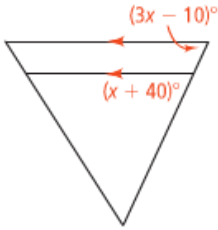


Algebra Find the value of x. Then find the measure of each labeled angle. See Problem 4.

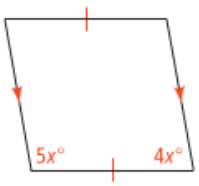
15.



16.



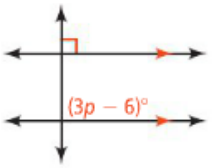
17.



B Apply

Algebra Find the values of the variables.

18.



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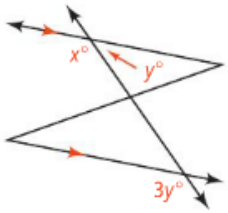
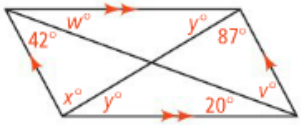


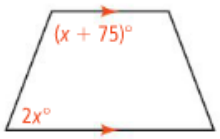
20.



21. Think About a Plan People in ancient Rome played a game called *terni lapilli*. The exact rules of this game are not known. Etchings on floors and walls in Rome suggest that the game required a grid of two intersecting pairs of parallel lines, similar to the modern game tick-tack-toe. The measure of one of the angles formed by the intersecting lines is 90°. Find the measure of each of the other 15 angles. Justify your answers.

- How can you use a diagram to help?
- You know the measure of one angle. How does the position of that angle relate to the position of each of the other angles?
- Which angles formed by two parallel lines and a transversal are congruent? Which angles are supplementary?

22. Error Analysis Which solution for the value of x in the figure at the right is incorrect? Explain.



A.

$$2x = x + 75$$
$$x = 75$$

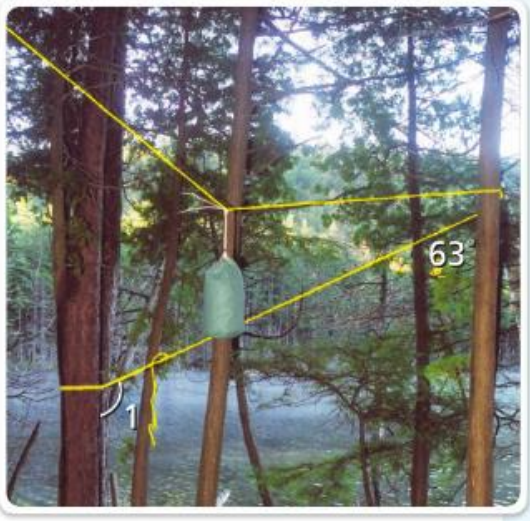
B.

$$2x + (x + 75) = 180$$
$$3x + 75 = 180$$
$$3x = 105$$
$$x = 35$$

23. Outdoor Recreation Campers often use a "bear bag" at night to avoid attracting animals to their food supply. In the bear bag system at the right, a camper pulls one end of the rope to raise and lower the food bag.

a. Suppose a camper pulls the rope taut between the two parallel trees, as shown. What is $m\angle 1$?

b. Are $\angle 1$ and the given angle *alternate interior angles*, *same-side interior angles*, or *corresponding angles*?

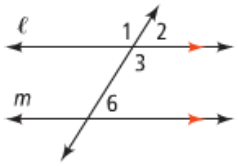


24. Writing Are same-side interior angles ever congruent? Explain.

25. Proof Write a two-column proof to prove the Same-Side Interior Angles Theorem (Theorem 3-2).

Given: $\ell \parallel m$

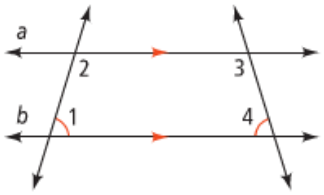
Prove: $\angle 3$ and $\angle 6$ are supplementary.



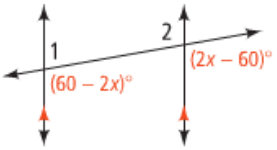
26. Proof Write a two-column proof.

Given: $a \parallel b$, $\angle 1 \cong \angle 4$

Prove: $\angle 2 \cong \angle 3$



Challenge **Use the diagram at the right for Exercises 27 and 28.**



27. **Algebra** Suppose the measures of $\angle 1$ and $\angle 2$ are in a $4 : 11$ ratio. Find their measures. (Diagram is not to scale.)

28. **Error Analysis** The diagram contains contradictory information. What is it? Why is it contradictory?

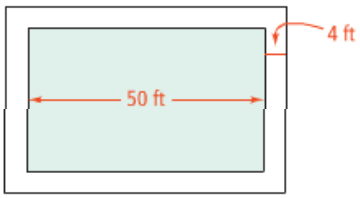
Standardized Test Prep

GRIDDED RESPONSE

SAT/ACT

29. $\angle 1$ and $\angle 2$ are same-side interior angles formed by two parallel lines and a transversal. If $m\angle 1 = 115$, what is $m\angle 2$?

30. The rectangular swimming pool shown at the right has an area of 1500 ft^2. A rectangular walkway surrounds the pool. How many feet of fencing do you need to surround the walkway?



31. The measure of an angle is two times the measure of its complement. What is the measure of the angle?

32. $\angle 1$ and $\angle 2$ are vertical angles. If $m\angle 1 = 4x$ and $m\angle 2 = 56$, what is the value of x?

Mixed Review

Determine whether each statement is *always*, *sometimes*, or *never* true. See Lesson 3-1.

33. Skew lines are coplanar.

34. Skew lines intersect.

35. Parallel planes intersect.

36. Rays are parallel.

Get Ready! **To prepare for Lesson 3-3, do Exercises 37–39.**

Write the converse and determine its truth value. See Lesson 2-2.

37. If a triangle is a right triangle, then it has a 90° angle.

38. If two angles are vertical angles, then they are congruent.

39. If two angles are same-side interior angles, then they are supplementary.

3-3 Proving Lines Parallel

Objective To determine whether two lines are parallel

You don't need a protractor. You can use what you learned in Lesson 3-2 to solve this maze problem.

SOLVE IT!

Getting Ready!

The maze below has two intersecting sets of parallel paths. A mouse makes five turns in the maze to get to a piece of cheese. Follow the mouse's path through the maze. What are the number of degrees at each turn? Explain how you know.

Lesson Vocabulary
- flow proof

In the Solve It, you used parallel lines to find congruent and supplementary relationships of special angle pairs. In this lesson you will do the converse. You will use the congruent and supplementary relationships of the special angle pairs to prove lines parallel.

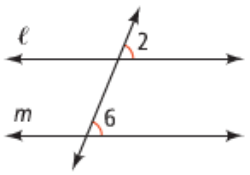
Essential Understanding You can use certain angle pairs to decide whether two lines are parallel.

take note

Postulate 3-2 Converse of the Corresponding Angles Postulate

Postulate
If two lines and a transversal form corresponding angles that are congruent, then the lines are parallel.

If . . .
$\angle 2 \cong \angle 6$



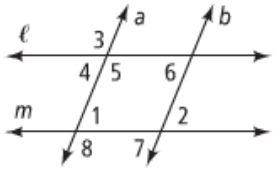
Then . . .
$\ell \parallel m$

Think

Which line is the transversal for ∠1 and ∠2?
Line m is the transversal because it forms one side of both angles.

Problem 1 Identifying Parallel Lines

Which lines are parallel if ∠1 ≅ ∠2? Justify your answer.



∠1 and ∠2 are corresponding angles. If ∠1 ≅ ∠2, then $a \parallel b$ by the Converse of the Corresponding Angles Postulate.

Got It? 1. Which lines are parallel if ∠6 ≅ ∠7? Justify your answer.

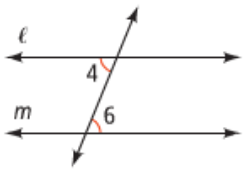
In Lesson 3-2 you proved theorems based on the Corresponding Angles Postulate. You can use the Converse of the Corresponding Angles Postulate to prove converses of the theorems you learned in Lesson 3-2.

take note

Theorem 3-4 Converse of the Alternate Interior Angles Theorem

Theorem
If two lines and a transversal form alternate interior angles that are congruent, then the two lines are parallel.

If . . .
∠4 ≅ ∠6

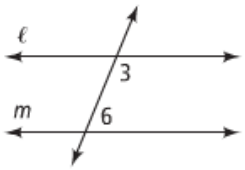


Then . . .
$\ell \parallel m$

Theorem 3-5 Converse of the Same-Side Interior Angles Theorem

Theorem
If two lines and a transversal form same-side interior angles that are supplementary, then the two lines are parallel.

If . . .
$m\angle 3 + m\angle 6 = 180$



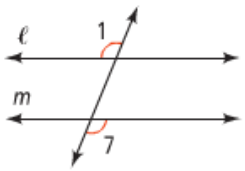
Then . . .
$\ell \parallel m$

You will prove Theorem 3-5 in Exercise 29.

Theorem 3-6 Converse of the Alternate Exterior Angles Theorem

Theorem
If two lines and a transversal form alternate exterior angles that are congruent, then the two lines are parallel.

If . . .
∠1 ≅ ∠7



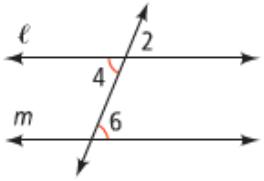
Then . . .
$\ell \parallel m$

The proof of the Converse of the Alternate Interior Angles Theorem below looks different than any proof you have seen so far in this course. You know two forms of proof—paragraph and two-column. In a third form, called **flow proof**, arrows show the logical connections between the statements. Reasons are written below the statements.

Proof Proof of Theorem 3-4: Converse of the Alternate Interior Angles Theorem

Given: $\angle 4 \cong \angle 6$

Prove: $\ell \parallel m$



$\angle 4 \cong \angle 6$ (Given)

$\angle 2 \cong \angle 4$ (Vertical $\measuredangle$ are $\cong$.)

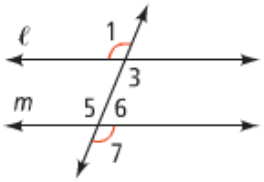
$\angle 2 \cong \angle 6$ (Transitive Property of $\cong$)

$\ell \parallel m$ (If corresp. $\measuredangle$ are $\cong$, then the lines are $\parallel$.)

Proof Problem 2 Writing a Flow Proof of Theorem 3-6

Given: $\angle 1 \cong \angle 7$

Prove: $\ell \parallel m$



Know

- $\angle 1 \cong \angle 7$

From the diagram you know

- $\angle 1$ and $\angle 3$ are vertical
- $\angle 5$ and $\angle 7$ are vertical
- $\angle 1$ and $\angle 5$ are corresponding
- $\angle 3$ and $\angle 7$ are corresponding

Need

One pair of corresponding angles congruent to prove $\ell \parallel m$

Plan

Use a pair of congruent vertical angles to relate either $\angle 1$ or $\angle 7$ to its corresponding angle.

$\angle 1 \cong \angle 7$ (Given)

$\angle 3 \cong \angle 1$ (Vertical $\measuredangle$ are $\cong$.)

$\angle 3 \cong \angle 7$ (Transitive Property of $\cong$)

$\ell \parallel m$ (If corresp. $\measuredangle$ are $\cong$, then the lines are $\parallel$.)

Got It? **2.** Use the same diagram and given information from Problem 2. Prove that $\angle 3 \cong \angle 5$ using a flow proof.

The postulate and three theorems you have just learned provide you with four ways to determine if two lines are parallel.

Problem 3 Determining Whether Lines are Parallel

Think

How do ∠1 and ∠2 relate to each other in the diagram?
∠1 and ∠2 are both exterior angles and they lie on opposite sides of *t*.

The fence gate at the right is made up of pieces of wood arranged in various directions. Suppose ∠1 ≅ ∠2. Are lines *r* and *s* parallel? Explain.

Yes, $r \parallel s$. ∠1 and ∠2 are alternate exterior angles. If two lines and a transversal form congruent alternate exterior angles, then the lines are parallel (Converse of the Alternate Exterior Angles Theorem).

Got It? **3.** In Problem 3, what is another way to explain why $r \parallel s$? Justify your answer.

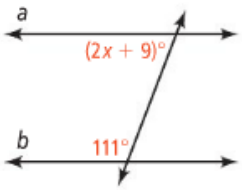
You can use algebra along with the postulates and theorems from Lesson 3-2 and Lesson 3-3 to help you solve problems involving parallel lines.

Problem 4 Using Algebra

Think

Work backward.
Think about what must be true of the given angles for *a* and *b* to be parallel.

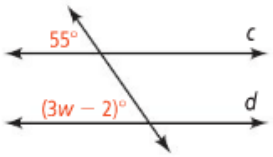
Algebra **What is the value of *x* for which $a \parallel b$?**



The two angles are same-side interior angles. By the Converse of the Same-Side Interior Angles Theorem, $a \parallel b$ if the angles are supplementary.

$$(2x + 9) + 111 = 180 \quad \text{Def. of supplementary angles}$$
$$2x + 120 = 180 \quad \text{Simplify.}$$
$$2x = 60 \quad \text{Subtract 120 from each side.}$$
$$x = 30 \quad \text{Divide each side by 2.}$$

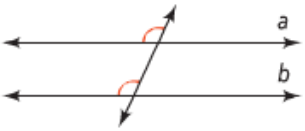
Got It? **4.** What is the value of *x* for which $c \parallel d$?

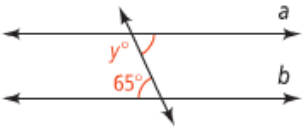


Lesson Check

Do you know HOW?

State the theorem or postulate that proves $a \parallel b$.

1. 

2. 

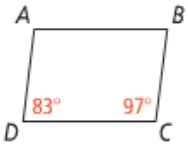
3. What is the value of y for which $a \parallel b$ in Exercise 2?

Do you UNDERSTAND?

4. Explain how you know when to use the Alternate Interior Angles Theorem and when to use the Converse of the Alternate Interior Angles Theorem.

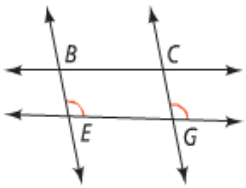
5. **Compare and Contrast** How are flow proofs and two-column proofs alike? How are they different?

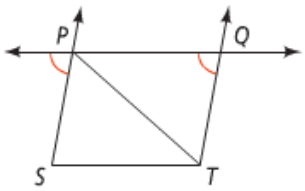
6. **Error Analysis** A classmate says that $\overleftrightarrow{AB} \parallel \overleftrightarrow{DC}$ based on the diagram at the right. Explain your classmate's error.

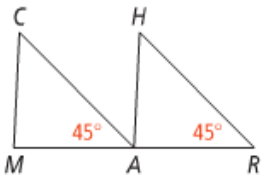


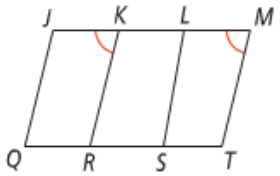
Practice and Problem-Solving Exercises

A Practice Which lines or segments are parallel? Justify your answer. See Problem 1.

7. 

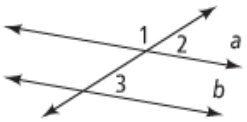
8. 

9. 

10. 

11. **Developing Proof** Complete the flow proof below. See Problem 2.

Given: $\angle 1$ and $\angle 3$ are supplementary.
Prove: $a \parallel b$



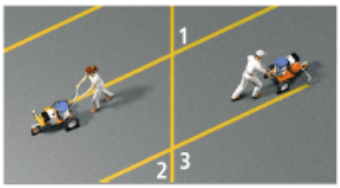
$\angle 1$ and $\angle 3$ are supplementary. → **d.** ? → $a \parallel b$

a. ? — Supplements of the same $\angle$ are $\cong$. — **e.** ?

b. ? → $\angle 1$ and $\angle 2$ are supplementary.

Def. of linear pair — **c.** ?

12. Parking Two workers paint lines for angled parking spaces. One worker paints a line so that $m\angle 1 = 65$. The other worker paints a line so that $m\angle 2 = 65$. Are their lines parallel? Explain.

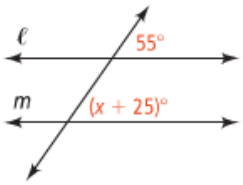


See Problem 3.

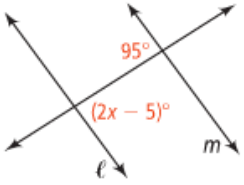
Algebra Find the value of x for which $\ell \parallel m$.

See Problem 4.

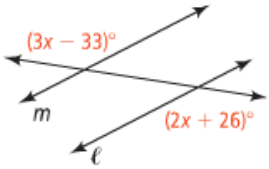
13.



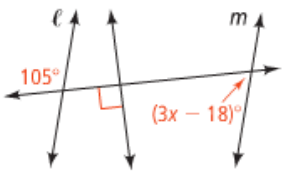
14.



15.

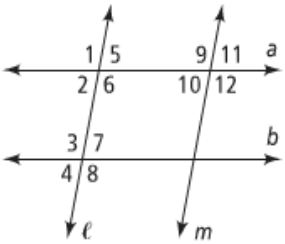


16.



B Apply

Developing Proof Use the given information to determine which lines, if any, are parallel. Justify each conclusion with a theorem or postulate.



17. $\angle 2$ is supplementary to $\angle 3$.

18. $\angle 1 \cong \angle 3$

19. $\angle 6$ is supplementary to $\angle 7$.

20. $\angle 9 \cong \angle 12$

21. $m\angle 7 = 65$, $m\angle 9 = 115$

22. $\angle 2 \cong \angle 10$

23. $\angle 1 \cong \angle 8$

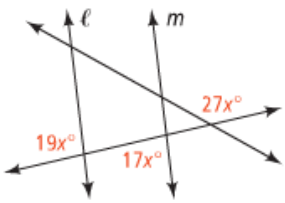
24. $\angle 8 \cong \angle 6$

25. $\angle 11 \cong \angle 7$

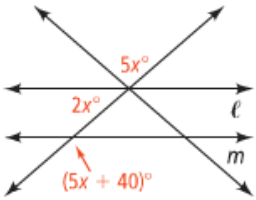
26. $\angle 5 \cong \angle 10$

Algebra Find the value of x for which $\ell \parallel m$.

27.



28.

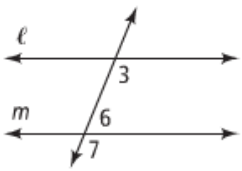


29. Prove the Converse of the Same-Side Interior Angles Theorem (Theorem 3-5).

Proof

Given: $m\angle 3 + m\angle 6 = 180$

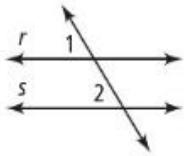
Prove: $\ell \parallel m$



30. Think About a Plan If the rowing crew at the right strokes in unison, the oars sweep out angles of equal measure. Explain why the oars on each side of the shell stay parallel.

- What type of information do you need to prove lines parallel?
- How do the positions of the angles of equal measure relate?

Algebra Determine the value of x for which $r \parallel s$. Then find $m\angle 1$ and $m\angle 2$.



31. $m\angle 1 = 80 - x, m\angle 2 = 90 - 2x$

32. $m\angle 1 = 60 - 2x, m\angle 2 = 70 - 4x$

33. $m\angle 1 = 40 - 4x, m\angle 2 = 50 - 8x$

34. $m\angle 1 = 20 - 8x, m\angle 2 = 30 - 16x$

Use the diagram at the right below for Exercises 35–41.

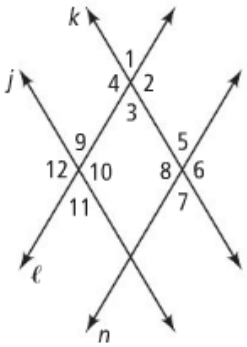
Open-Ended Use the given information. State another fact about one of the given angles that will guarantee two lines are parallel. Tell which lines will be parallel and why.

35. $\angle 1 \cong \angle 3$

36. $m\angle 8 = 110, m\angle 9 = 70$

37. $\angle 5 \cong \angle 11$

38. $\angle 11$ and $\angle 12$ are supplementary.



39. Reasoning If $\angle 1 \cong \angle 7$, what theorem or postulate can you use to show that $\ell \parallel n$?

Write a flow proof.

40. Proof **Given:** $\ell \parallel n, \angle 12 \cong \angle 8$
Prove: $j \parallel k$

41. Proof **Given:** $j \parallel k, m\angle 8 + m\angle 9 = 180$
Prove: $\ell \parallel n$

Challenge

Which sides of quadrilateral *PLAN* must be parallel? Explain.

42. $m\angle P = 72, m\angle L = 108, m\angle A = 72, m\angle N = 108$

43. $m\angle P = 59, m\angle L = 37, m\angle A = 143, m\angle N = 121$

44. $m\angle P = 67, m\angle L = 120, m\angle A = 73, m\angle N = 100$

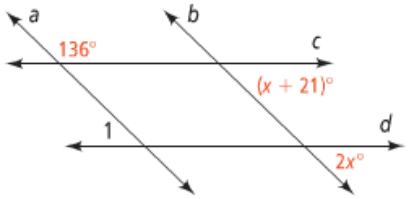
45. $m\angle P = 56, m\angle L = 124, m\angle A = 124, m\angle N = 56$

46. Proof Write a two-column proof to prove the following: If a transversal intersects two parallel lines, then the bisectors of two corresponding angles are parallel. (*Hint:* Start by drawing and marking a diagram.)

Standardized Test Prep

SAT/ACT

Use the diagram for Exercises 47 and 48.



47. For what value of x is $c \parallel d$?

Ⓐ 21 Ⓒ 43

Ⓑ 23 Ⓓ 53

48. If $c \parallel d$, what is $m\angle 1$?

Ⓕ 24 Ⓗ 136

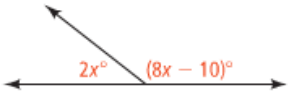
Ⓖ 44 Ⓘ 146

49. Which of the following is always a valid conclusion for the hypothesis?
If two angles are congruent, then ___?___.

Ⓐ they are right angles Ⓒ they have the same measure

Ⓑ they share a vertex Ⓓ they are acute angles

50. What is the value of x in the diagram at the right?



Ⓕ $1.\overline{6}$ Ⓗ 17

Ⓖ 10 Ⓘ 19

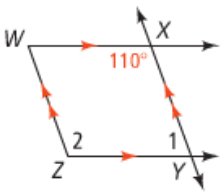
Short Response

51. Draw a pentagon. Is your pentagon convex or concave? Explain.

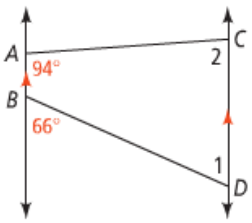
Mixed Review

Find $m\angle 1$ and $m\angle 2$. Justify each answer. See Lesson 3-2.

52.



53.



Get Ready! **To prepare for Lesson 3-4, do Exercises 54–57.**

Determine whether each statement is *always*, *sometimes*, or *never* true. See Lessons 1-6 and 3-1.

54. Perpendicular lines meet at right angles.

55. Two lines in intersecting planes are perpendicular.

56. Two lines in the same plane are parallel.

57. Two lines in parallel planes are perpendicular.

3-4 Parallel and Perpendicular Lines

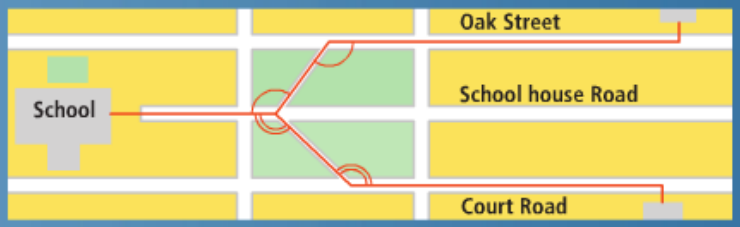
Objective To relate parallel and perpendicular lines

Look at the angle markings. What do they tell you?

SOLVE IT!

Getting Ready!

Jude and Jasmine leave school together to walk home. Then Jasmine cuts down a path from Schoolhouse Road to get to Oak Street and Jude cuts down another path to get to Court Road. Below is a diagram of the route each follows home. What conjecture can you make about Oak Street and Court Road? Explain.



In the Solve It, you likely made your conjecture about Oak Street and Court Road based on their relationships to Schoolhouse Road. In this lesson you will use similar reasoning to prove that lines are parallel or perpendicular.

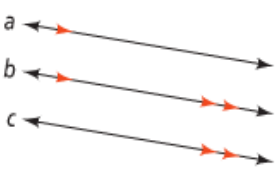
Essential Understanding You can use the relationships of two lines to a third line to decide whether the two lines are parallel or perpendicular to each other.

take note

Theorem 3-7

Theorem
If two lines are parallel to the same line, then they are parallel to each other.

If . . .
$a \parallel b$ and $b \parallel c$



Then . . .
$a \parallel c$

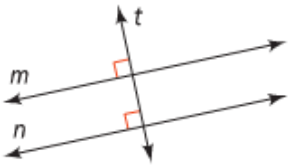
You will prove Theorem 3-7 in Exercise 7.

take note

Theorem 3-8

Theorem
In a plane, if two lines are perpendicular to the same line, then they are parallel to each other.

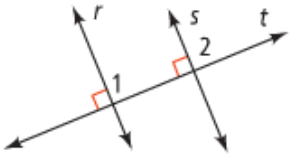
If . . .
$m \perp t$ and $n \perp t$



Then . . .
$m \parallel n$

Notice that Theorem 3-8 includes the phrase *in a plane*. In Exercise 17, you will consider why this phrase is necessary.

Proof Proof of Theorem 3-8



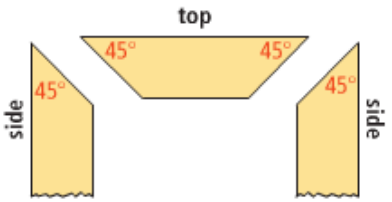
Given: In a plane, $r \perp t$ and $s \perp t$.

Prove: $r \parallel s$

Proof: $\angle 1$ and $\angle 2$ are right angles by the definition of perpendicular. So, $\angle 1 \cong \angle 2$. Since corresponding angles are congruent, $r \parallel s$.

Problem 1 Solving a Problem With Parallel Lines

Carpentry **A carpenter plans to install molding on the sides and the top of a doorway. The carpenter cuts the ends of the top piece and one end of each of the side pieces at 45° angles as shown. Will the side pieces of molding be parallel? Explain.**



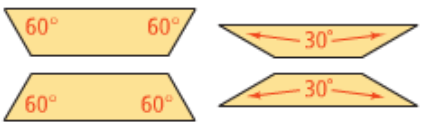
Know
The angles at the connecting ends are 45°.

Need
Determine whether the side pieces of molding are parallel.

Plan
Visualize fitting the pieces together to form new angles. Use information about the new angles to decide whether the sides are parallel.

Yes, the sides are parallel. When the pieces fit together, they form 45° + 45°, or 90°, angles. So, each side is perpendicular to the top. If two lines (the sides) are perpendicular to the same line (the top), then they are parallel to each other.

Got It? **1.** Can you assemble the pieces at the right to form a picture frame with opposite sides parallel? Explain.



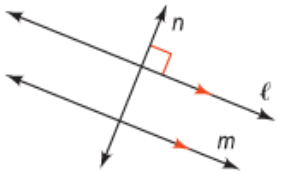
Theorems 3-7 and 3-8 give conditions that allow you to conclude that lines are parallel. The Perpendicular Transversal Theorem below provides a way for you to conclude that lines are perpendicular.

take note

Theorem 3-9 Perpendicular Transversal Theorem

Theorem
In a plane, if a line is perpendicular to one of two parallel lines, then it is also perpendicular to the other.

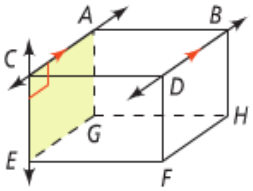
If . . .
$n \perp \ell$ and $\ell \parallel m$



Then . . .
$n \perp m$

You will prove Theorem 3-9 in Exercise 10.

The Perpendicular Transversal Theorem states that the lines must be *in a plane*. The diagram at the right shows why. In the rectangular solid, $\overleftrightarrow{AC}$ and $\overleftrightarrow{BD}$ are parallel. $\overleftrightarrow{EC}$ is perpendicular to $\overleftrightarrow{AC}$, but it is not perpendicular to $\overleftrightarrow{BD}$. In fact, $\overleftrightarrow{EC}$ and $\overleftrightarrow{BD}$ are skew because they are not in the same plane.



Proof

Problem 2 Proving a Relationship Between Two Lines

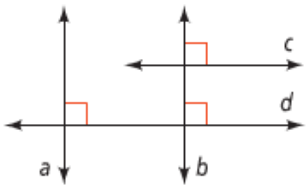
Think

Which line has a relationship to both lines *c* and *d*?
You know $c \perp b$ and $b \perp d$. So, line b relates to both lines c and d.

Given: In a plane, $c \perp b$, $b \perp d$, and $d \perp a$.

Prove: $c \perp a$

Proof: Lines c and d are both perpendicular to line b, so $c \parallel d$ because two lines perpendicular to the same line are parallel. It is given that $d \perp a$. Therefore, $c \perp a$ because a line that is perpendicular to one of two parallel lines is also perpendicular to the other (Perpendicular Transversal Theorem).



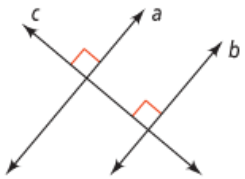
Got It? **2.** In Problem 2, could you also conclude $a \parallel b$? Explain.

Lesson Check

Do you know HOW?

1. Main Street intersects Avenue A and Avenue B. Avenue A is parallel to Avenue B. Avenue A is also perpendicular to Main Street. How are Avenue B and Main Street related? Explain.

2. In the diagram below, lines a, b, and c are coplanar. What conclusion can you make about lines a and b? Explain.

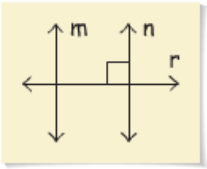


Do you UNDERSTAND?

3. Explain why the phrase *in a plane* is not necessary in Theorem 3-7.

4. Which theorem or postulate from earlier in the chapter supports the conclusion in Theorem 3-8? In the Perpendicular Transversal Theorem? Explain.

5. **Error Analysis** Shiro sketched coplanar lines m, n, and r on his homework paper. He claims that it shows that lines m and n are parallel. What other information do you need about line r in order for Shiro's claim to be true? Explain.

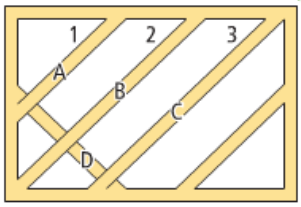


Practice and Problem-Solving Exercises

A Practice

6. A carpenter is building a trellis for vines to grow on. The completed trellis will have two sets of diagonal pieces of wood that overlap each other.
   - **a.** If pieces A, B, and C must be parallel, what must be true of $\angle 1$, $\angle 2$, and $\angle 3$?
   - **b.** The carpenter attaches piece D so that it is perpendicular to piece A. If your answer to part (a) is true, is piece D perpendicular to pieces B and C? Justify your answer.

See Problem 1.



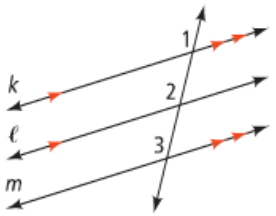
7. **Developing Proof** Copy and complete this paragraph proof of Theorem 3-7 for three coplanar lines.

See Problem 2.

Given: $\ell \parallel k$ and $m \parallel k$

Prove: $\ell \parallel m$

Proof: Since $\ell \parallel k$, $\angle 2 \cong \angle 1$ by the **a.** ? Postulate. Since $m \parallel k$, **b.** ? $\cong$ **c.** ? for the same reason. By the Transitive Property of Congruence, $\angle 2 \cong \angle 3$. By the **d.** ? Postulate, $\ell \parallel m$.

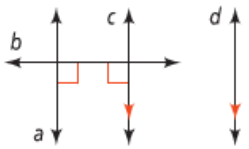


8. Write a paragraph proof.

Proof

Given: In a plane, $a \perp b$, $b \perp c$, and $c \parallel d$.

Prove: $a \parallel d$

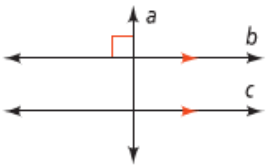


B Apply

9. Think About a Plan One traditional type of log cabin is a single rectangular room. Suppose you begin building a log cabin by placing four logs in the shape of a rectangle. What should you measure to guarantee that the logs on opposite walls are parallel? Explain.

- What type of information do you need to prove lines parallel?
- How can you use a diagram to help you?
- What do you know about the angles of the geometric shape?

10. Proof Prove the Perpendicular Transversal Theorem (Theorem 3-9): In a plane, if a line is perpendicular to one of two parallel lines, then it is also perpendicular to the other.



Given: In a plane, $a \perp b$ and $b \parallel c$.

Prove: $a \perp c$

The following statements describe a ladder. Based only on the statement, make a conclusion about the rungs, one side, or both sides of the ladder. Explain.

11. The rungs are each perpendicular to one side.

12. The rungs are parallel and the top rung is perpendicular to one side.

13. The sides are parallel. The rungs are perpendicular to one side.

14. Each side is perpendicular to the top rung.

15. The rungs are perpendicular to one side. The sides are not parallel.

16. Public Transportation The map at the right is a section of a subway map. The yellow line is perpendicular to the brown line, the brown line is perpendicular to the blue line, and the blue line is perpendicular to the pink line. What conclusion can you make about the yellow line and the pink line? Explain.

17. Writing Theorem 3-8 states that in a plane, two lines perpendicular to the same line are parallel. Explain why the phrase *in a plane* is needed. (*Hint:* Refer to a rectangular solid to help you visualize the situation.)

18. Quilting You plan to sew two triangles of fabric together to make a square for a quilting project. The triangles are both right triangles and have the same side and angle measures. What must also be true about the triangles in order to guarantee that the opposite sides of the fabric square are parallel? Explain.

C Challenge

For Exercises 19–24, *a*, *b*, *c*, and *d* are distinct lines in the same plane. For each combination of relationships, tell how *a* and *d* relate. Justify your answer.

19. $a \parallel b, b \parallel c, c \parallel d$

20. $a \parallel b, b \parallel c, c \perp d$

21. $a \parallel b, b \perp c, c \parallel d$

22. $a \perp b, b \parallel c, c \parallel d$

23. $a \parallel b, b \perp c, c \perp d$

24. $a \perp b, b \parallel c, c \perp d$

25. Reasoning Review the reflexive, symmetric, and transitive properties for congruence in Lesson 2-5. Write reflexive, symmetric, and transitive statements for "is parallel to" ($\parallel$). Tell whether each statement is *true* or *false*. Justify your answer.

26. Reasoning Repeat Exercise 25 for "is perpendicular to" ($\perp$).

Standardized Test Prep

SAT/ACT

27. In a plane, line e is parallel to line f, line f is parallel to line g, and line h is perpendicular to line e. Which of the following MUST be true?

(A) $e \parallel g$ (B) $h \parallel f$ (C) $g \parallel h$ (D) $e \parallel h$

28. Which point lies nearest to $(5, 2)$ in the coordinate plane?

(F) $(-1, 3)$ (G) $(0, -2)$ (H) $(4, -5)$ (I) $(4, 10)$

29. Which of the following is NOT a reason for proving two lines parallel.

(A) The lines are both $\perp$ to the same line. (C) Vertical angles are congruent.

(B) Corresponding angles are congruent. (D) The lines are both $\parallel$ to the same line.

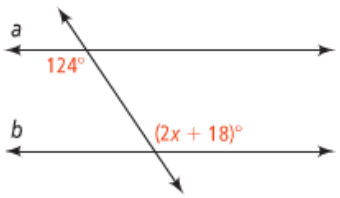
Short Response

30. The diameter of a circle is the same length as the side of a square. The perimeter of the square is 16 cm. Find the diameter of the circle. Then find the circumference of the circle in terms of π.

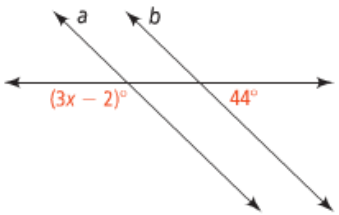
Mixed Review

Algebra Determine the value of x for which $a \parallel b$. See Lesson 3-3.

31.



32.



Use a protractor. Classify each angle as *acute*, *right*, or *obtuse*. See Lesson 1-4.

33.

34.

35.

Get Ready! **To prepare for Lesson 3-5, do Exercises 36–39.**

Solve each equation. See p. 834.

36. $30 + 90 + x = 180$

37. $55 + x + 105 = 180$

38. $x + 50 = 90$

39. $32 + x = 90$

Concept Byte

Use With Lesson 3-4

ACTIVITY

Perpendicular Lines and Planes

As you saw in Chapter 1, you can use a polygon to represent a plane in space. You can sketch overlapping polygons to suggest how two perpendicular planes intersect in a line.

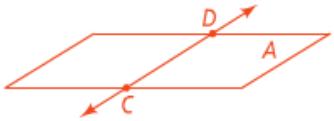
Activity

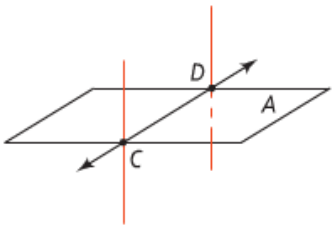
Draw perpendicular planes A and B intersecting in $\overleftrightarrow{CD}$.

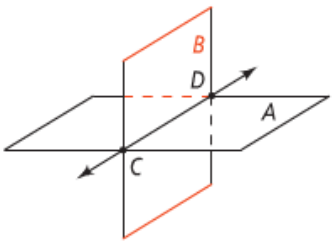
Step 1 Draw plane A and $\overleftrightarrow{CD}$ in plane A.

Step 2 Draw two segments that are perpendicular to $\overleftrightarrow{CD}$. One segment should pass through point C. The other segment should pass through point D. The segments represent two lines in plane B that are perpendicular to plane A.

Step 3 Connect the segment endpoints to draw plane B. Plane B is perpendicular to plane A because plane B contains lines perpendicular to plane A.







Exercises

1. Draw a plane in space. Then draw two lines that are in the plane and intersect at point A. Draw a third line that is perpendicular to each of the two lines at point A. What is the relationship between the third line and the plane?

2. **a.** Draw a plane and a point in the plane. Draw a line perpendicular to the plane at that point. Can you draw more than one perpendicular line?
   b. Draw a line and a point on the line. Draw a plane that is perpendicular to the line at that point. Can you draw more than one perpendicular plane?

3. Draw two planes perpendicular to the same line. What is the relationship between the planes?

4. Draw line ℓ through plane P at point A, so that line ℓ is perpendicular to plane P.
   a. Draw line m perpendicular to line ℓ at point A. How do m and plane P relate? Does this relationship hold true for every line perpendicular to line ℓ at point A?
   b. Draw a plane Q that contains line ℓ. How do planes P and Q relate? Does this relationship hold true for every plane Q that contains line ℓ?

3-5 Parallel Lines and Triangles

Objectives To use parallel lines to prove a theorem about triangles
To find measures of angles of triangles

If you try another triangle, do you think you would get the same result?

Getting Ready!

Draw and cut out a large triangle. What is the sum of the angle measures of the triangle? Explain. Do not use a protractor. (Hint: Tear off and rearrange the three corners of the triangle.)

Dynamic Activity
Triangle Theorems Activity

Lesson Vocabulary
- auxiliary line
- exterior angle of a polygon
- remote interior angles

In the Solve It, you may have discovered that you can rearrange the corners of the triangle to form a straight angle. You can do this for any triangle.

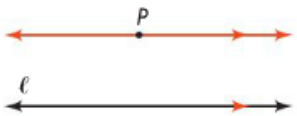
Essential Understanding The sum of the angle measures of a triangle is always the same.

The Solve It suggests an important theorem about triangles. To prove this theorem, you will need to use parallel lines.

take note

Postulate 3-3 Parallel Postulate

Through a point not on a line, there is one and only one line parallel to the given line.

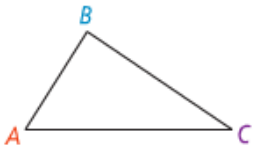


There is exactly one line through P parallel to ℓ.

take note

Theorem 3-10 Triangle Angle-Sum Theorem

The sum of the measures of the angles of a triangle is 180.



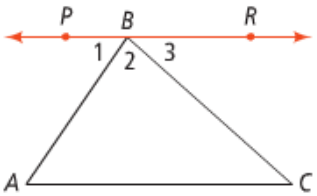
$m\angle A + m\angle B + m\angle C = 180$

The proof of the Triangle Angle-Sum Theorem requires an *auxiliary line*. An **auxiliary line** is a line that you add to a diagram to help explain relationships in proofs. The red line in the diagram below is an auxiliary line.

Proof Proof of Theorem 3-10: Triangle Angle-Sum Theorem

Given: $\triangle ABC$

Prove: $m\angle A + m\angle 2 + m\angle C = 180$

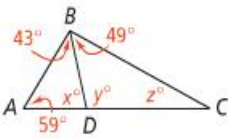


Statements	Reasons
1) Draw $\overleftrightarrow{PR}$ through B, parallel to $\overline{AC}$.	1) Parallel Postulate
2) $\angle PBC$ and $\angle 3$ are supplementary.	2) $\measuredangle$s that form a linear pair are suppl.
3) $m\angle PBC + m\angle 3 = 180$	3) Definition of suppl. $\measuredangle$s
4) $m\angle PBC = m\angle 1 + m\angle 2$	4) Angle Addition Postulate
5) $m\angle 1 + m\angle 2 + m\angle 3 = 180$	5) Substitution Property
6) $\angle 1 \cong \angle A$ and $\angle 3 \cong \angle C$	6) If lines are $\parallel$, then alternate interior $\measuredangle$s are $\cong$.
7) $m\angle 1 = m\angle A$ and $m\angle 3 = m\angle C$	7) Congruent $\measuredangle$s have equal measure.
8) $m\angle A + m\angle 2 + m\angle C = 180$	8) Substitution Property

When you know the measures of two angles of a triangle, you can use the Triangle Angle-Sum Theorem to find the measure of the third angle.

Problem 1 Using the Triangle Angle-Sum Theorem

Algebra What are the values of x and y in the diagram at the right?



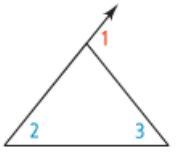
Plan

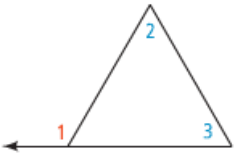
Which variable should you solve for first?
From the diagram, you know two angle measures in $\triangle ADB$. The third angle is labeled $x°$. So use what you know about the angle measures in a triangle to solve for x first.

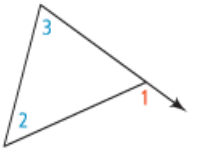
Think	Write
Use the Triangle Angle-Sum Theorem to write an equation involving x.	$59 + 43 + x = 180$
Solve for x by simplifying and then subtracting 102 from each side.	$102 + x = 180$ $x = 78$
$\angle ADB$ and $\angle CDB$ form a linear pair, so they are supplementary.	$m\angle ADB + m\angle CDB = 180$
Substitute 78 for $m\angle ADB$ and y for $m\angle CDB$ in the above equation.	$x + y = 180$ $78 + y = 180$
Solve for y by subtracting 78 from each side.	$y = 102$

Got It? **1.** Use the diagram in Problem 1. What is the value of z?

An **exterior angle of a polygon** is an angle formed by a side and an extension of an adjacent side. For each exterior angle of a triangle, the two nonadjacent interior angles are its **remote interior angles.** In each triangle below, $\angle 1$ is an exterior angle and $\angle 2$ and $\angle 3$ are its remote interior angles.





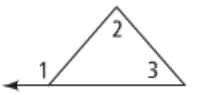


The theorem below states the relationship between an exterior angle and its two remote interior angles.

take note

Theorem 3-11 Triangle Exterior Angle Theorem

The measure of each exterior angle of a triangle equals the sum of the measures of its two remote interior angles.



$$m\angle 1 = m\angle 2 + m\angle 3$$

You will prove Theorem 3-11 in Exercise 33.

You can use the Triangle Exterior Angle Theorem to find angle measures.

Problem 2 Using the Triangle Exterior Angle Theorem

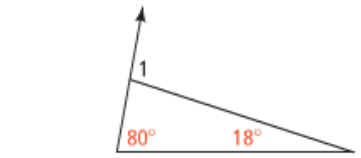
Plan

What information can you get from the diagram?
The diagram shows you which angles are interior or exterior.

A What is the measure of $\angle 1$?

$m\angle 1 = 80 + 18$ Triangle Exterior Angle Theorem

$m\angle 1 = 98$ Simplify.



B What is the measure of $\angle 2$?

$124 = 59 + m\angle 2$ Triangle Exterior Angle Theorem

$65 = m\angle 2$ Subtract 59 from each side.

Got It? **2.** Two angles of a triangle measure 53. What is the measure of an exterior angle at each vertex of the triangle?

Problem 3 Applying the Triangle Theorems

Plan

How can you apply your skills from Problem 2 here?
Look at the diagram. Notice that you have a triangle and information about interior and exterior angles.

Multiple Choice When radar tracks an object, the reflection of signals off the ground can result in clutter. Clutter causes the receiver to confuse the real object with its reflection, called a ghost. At the right, there is a radar receiver at A, an airplane at B, and the airplane's ghost at D. What is the value of x?

Ⓐ 30 Ⓒ 70

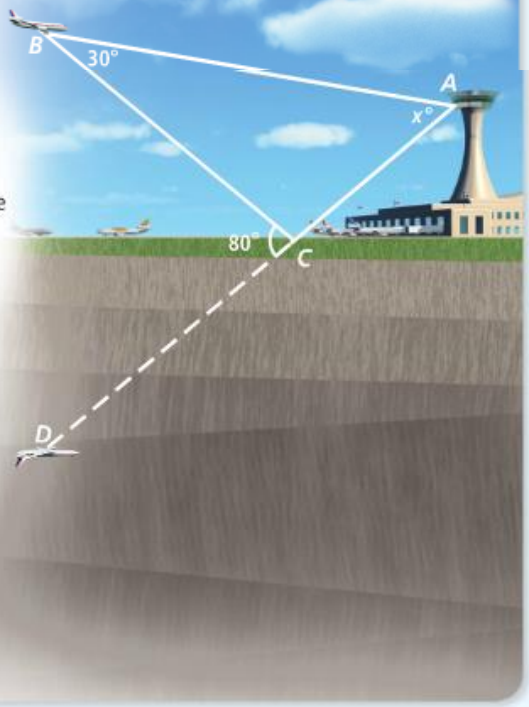
Ⓑ 50 Ⓓ 80

$m\angle A + m\angle B = m\angle BCD$ Triangle Exterior Angle Theorem

$x + 30 = 80$ Substitute.

$x = 50$ Subtract 30 from each side.

The value of x is 50. The correct answer is B.



Got It? **3. Reasoning** In Problem 3, can you find $m\angle A$ without using the Triangle Exterior Angle Theorem? Explain.

Lesson Check

Do you know HOW?

Find the measure of the third angle of a triangle given the measures of two angles.

1. 34 and 88
2. 45 and 90
3. 10 and 102
4. x and 50

In a triangle, $\angle 1$ is an exterior angle and $\angle 2$ and $\angle 3$ are its remote interior angles. Find the missing angle measure.

5. $m\angle 2 = 24$ and $m\angle 3 = 106$
6. $m\angle 1 = 70$ and $m\angle 2 = 32$

Do you UNDERSTAND?

7. Explain how the Triangle Exterior Angle Theorem makes sense based on the Triangle Angle-Sum Theorem.
8. **Error Analysis** The measures of the interior angles of a triangle are 30, x, and $3x$. Which of the following methods for solving for x is incorrect? Explain.

A.

$x + 3x = 30$
$4x = 30$
$x = 7.5$

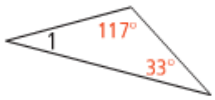
B.

$x + 3x + 30 = 180$
$4x + 30 = 180$
$4x = 150$
$x = 37.5$

Practice and Problem-Solving Exercises

A Practice

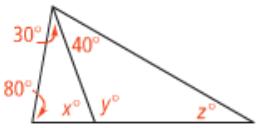
Find $m\angle 1$. See Problem 1.

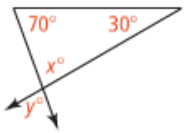
9. 

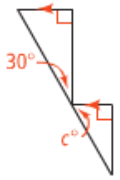
10.

11.

Algebra Find the value of each variable.

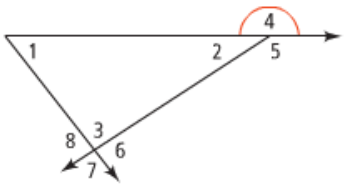
12. 

13. 

14. 

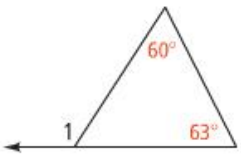
Use the diagram at the right for Exercises 15 and 16. See Problem 2.

15. **a.** Which of the numbered angles are exterior angles?
    b. Name the remote interior angles for each exterior angle.
    c. How are exterior angles 6 and 8 related?
16. **a.** How many exterior angles are at each vertex of the triangle?
    b. How many exterior angles does a triangle have in all?



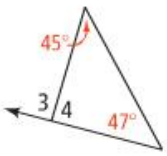
Algebra Find each missing angle measure.

17.

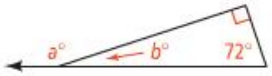


18.

19.



20. A ramp forms the angles shown at the right. What are the values of a and b?



See Problem 3.

21. A lounge chair has different settings that change the angles formed by its parts. Suppose $m\angle 2 = 71$ and $m\angle 3 = 43$. Find $m\angle 1$.

B Apply

Algebra Use the given information to find the unknown angle measures in the triangle.

22. The ratio of the angle measures of the acute angles in a right triangle is 1 : 2.

23. The measure of one angle of a triangle is 40. The measures of the other two angles are in a ratio of 3 : 4.

24. The measure of one angle of a triangle is 108. The measures of the other two angles are in a ratio of 1 : 5.

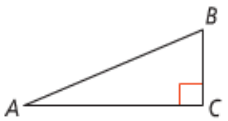
25. Think About a Plan The angle measures of $\triangle RST$ are represented by $2x$, $x + 14$, and $x - 38$. What are the angle measures of $\triangle RST$?

- How can you use the Triangle Angle-Sum Theorem to write an equation?
- How can you check your answer?

26. Proof Prove the following theorem: The acute angles of a right triangle are complementary.

Given: $\triangle ABC$ with right angle C

Prove: $\angle A$ and $\angle B$ are complementary.

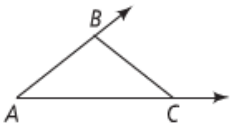


27. Reasoning What is the measure of each angle of an equiangular triangle? Explain.

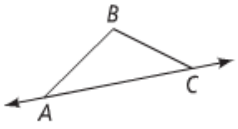
28. Draw a Diagram Which diagram below correctly represents the following description? Explain your reasoning.

Draw any triangle. Label it $\triangle ABC$. Extend two sides of the triangle to form two exterior angles at vertex A.

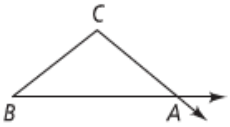
I.



II.

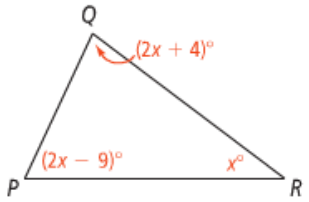


III.

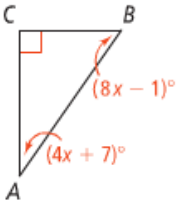


Find the values of the variables and the measures of the angles.

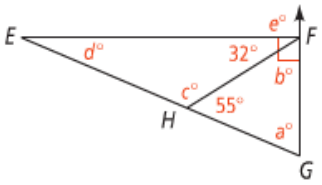
29.



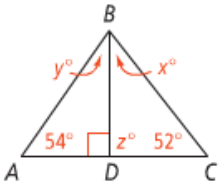
30.



31.



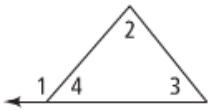
32.



33. Prove the Triangle Exterior Angle Theorem (Theorem 3-11).

Proof

The measure of each exterior angle of a triangle equals the sum of the measures of its two remote interior angles.



Given: $\angle 1$ is an exterior angle of the triangle.

Prove: $m\angle 1 = m\angle 2 + m\angle 3$

34. Reasoning Two angles of a triangle measure 64 and 48. What is the measure of the largest exterior angle of the triangle? Explain.

35. Algebra A right triangle has exterior angles at each of its acute angles with measures in the ratio 13 : 14. Find the measures of the two acute angles of the right triangle.

Challenge

Probability In Exercises 36–40, you know only the given information about the measures of the angles of a triangle. Find the probability that the triangle is equiangular.

36. Each is a multiple of 30.

37. Each is a multiple of 20.

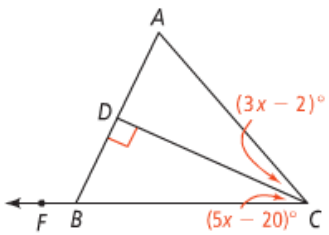
38. Each is a multiple of 60.

39. Each is a multiple of 12.

40. One angle is obtuse.

41. In the figure at the right, $\overline{CD} \perp \overline{AB}$ and $\overline{CD}$ bisects $\angle ACB$. Find $m\angle DBF$.

42. If the remote interior angles of an exterior angle of a triangle are congruent, what can you conclude about the bisector of the exterior angle? Justify your answer.



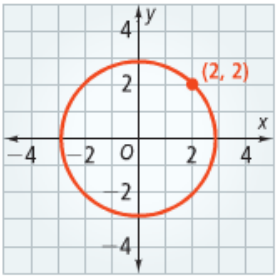
Standardized Test Prep

SAT/ACT

43. The measure of one angle of a triangle is 115. The other two angles are congruent. What is the measure of each of the congruent angles?

Ⓐ 32.5 Ⓑ 57.5 Ⓒ 65 Ⓓ 115

44. The center of the circle at the right is at the origin. What is the approximate length of its diameter?



Ⓕ 2 Ⓗ 5.6

Ⓖ 2.8 Ⓘ 8

45. One statement in a proof is "$\angle 1$ and $\angle 2$ are supplementary angles." The next statement is "$m\angle 1 + m\angle 2 = 180$." Which is the best justification for the second statement based on the first statement?

Ⓐ The sum of the measures of two right angles is 180.

Ⓑ Angles that form a linear pair are supplementary.

Ⓒ Definition of supplementary angles

Ⓓ The measure of a straight angle is 180.

Extended Response

46. $\triangle ABC$ is an obtuse triangle with $m\angle A = 21$ and $\angle C$ is acute.

a. What is $m\angle B + m\angle C$? Explain.

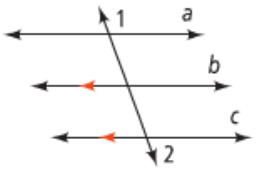
b. What is the range of whole numbers for $m\angle C$? Explain.

c. What is the range of whole numbers for $m\angle B$? Explain.

Mixed Review

Use the diagram at the right for Exercises 47 and 48.

See Lesson 3-4.



47. If $\angle 1$ and $\angle 2$ are supplementary, what can you conclude about lines a and c? Justify your answer.

48. If $a \parallel c$, what can you conclude about lines a and b? Justify your answer.

49. $\angle ABC$ and $\angle CBD$ form a linear pair. If $m\angle ABC = 3x + 20$ and $m\angle CBD = x + 32$, find the value of x.

See Lesson 1-5.

50. $\angle 1$ and $\angle 2$ are supplementary. If $\angle 1 \cong \angle 2$, find $m\angle 1$ and $m\angle 2$. Explain.

Get Ready! **To prepare for Lesson 3-6, do Exercises 51–53.**

Use a straightedge to draw each figure. Then use a straightedge and compass to construct a figure congruent to it.

See Lesson 1-6.

51. a segment **52.** an obtuse angle **53.** an acute angle

Concept Byte

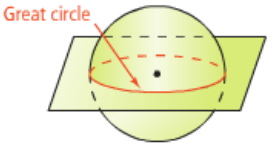
Use With Lesson 3-5

ACTIVITY

Exploring Spherical Geometry

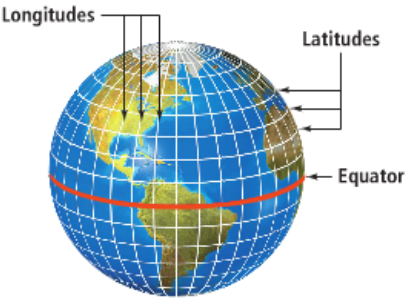
Euclid was a Greek mathematician who identified many of the definitions, postulates, and theorems of high school geometry. Euclidean geometry is the geometry of flat planes, straight lines, and points.

In spherical geometry, the curved surface of a sphere is studied. A "line" is a great circle. A *great circle* is the intersection of a sphere and a plane that contains the center of the sphere.



Activity 1

You can use latitude and longitude to identify positions on Earth. Look at the latitude and longitude markings on the globe.

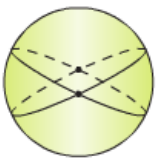


1. Think about "slicing" the globe with a plane at each latitude. Do any of your "slices" contain the center of the globe?
2. Think about "slicing" the globe with a plane at each longitude. Do any of your "slices" contain the center of the globe?
3. Which latitudes, if any, suggest great circles? Which longitudes, if any, suggest great circles? Explain.

You learned in Lesson 3.5 that through any point not on a line, there is one and only one line parallel to the given line (Parallel Postulate). That statement is not true in spherical geometry. In spherical geometry,

through a point not on a line, there is no line parallel to the given line.

Since lines are great circles in spherical geometry, two lines always intersect. In fact, any two lines on a sphere intersect at *two* points, as shown at the right.

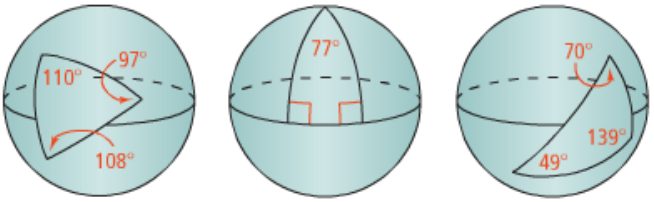


One result of the Parallel Postulate in Euclidean geometry is the Triangle Angle-Sum Theorem. The spherical geometry Parallel Postulate gives a very different result.

Activity 2

Hold a string taut between any two points on a sphere. The string forms a "segment" that is part of a great circle. Connect three such segments to form a triangle on the sphere.

Below are examples of triangles on a sphere.



4. What is the sum of the angle measures in the first triangle? The second triangle? The third triangle?

5. How are these results different from the Triangle Angle-Sum Theorem in Euclidean geometry? Explain.

Exercises

For Exercises 6 and 7, draw a sketch to illustrate each property of spherical geometry. Explain how each property compares to what is true in Euclidean geometry.

6. There are pairs of points on a sphere through which you can draw more than one line.

7. Two equiangular triangles can have different angle measures.

8. For each of the following properties of Euclidean geometry, draw a counterexample to show that the property is *not* true in spherical geometry.
   a. Two lines that are perpendicular to the same line do not intersect.
   b. If two angles of one triangle are congruent to two angles of another triangle, then the third angles are congruent.

9. a. The figure at the right appears to show parallel lines on a sphere. Explain why this is not the case.
   b. Explain why a piece of the top circle in the figure is *not* a line segment. (*Hint:* What must be true of line segments in spherical geometry?)

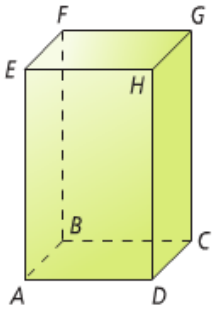
10. In Euclidean geometry, vertical angles are congruent. Does this seem to be true in spherical geometry? Explain. Make figures on a globe, ball, or balloon to support your answer.

3 Mid-Chapter Quiz

MathXL® for School
Go to PowerGeometry.com

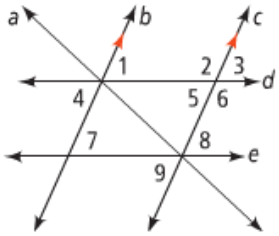
Do you know HOW?

Identify the following. Lines and planes that appear to be parallel are parallel.



1. all segments parallel to $\overline{HG}$
2. a plane parallel to plane EFB
3. all segments skew to $\overline{EA}$
4. all segments parallel to plane $ABCD$

Use the diagram below for Exercises 5–14.



Name two pairs of each angle type.

5. corresponding angles
6. alternate interior angles
7. same-side interior angles

State the theorem or postulate that justifies each statement.

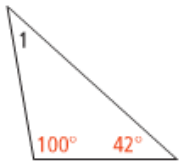
8. $\angle 7 \cong \angle 9$
9. $\angle 4 \cong \angle 5$
10. $m\angle 1 + m\angle 2 = 180$

Complete each statement.

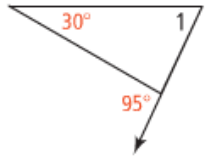
11. If $\angle 5 \cong \angle 9$, then $\underline{?} \parallel \underline{?}$.
12. If $\angle 4 \cong \underline{?}$, then $d \parallel e$.
13. If $e \perp b$, then $e \perp \underline{?}$.
14. If $c \perp d$, then $b \perp \underline{?}$.

Find $m\angle 1$.

15.

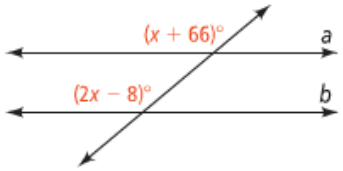


16.

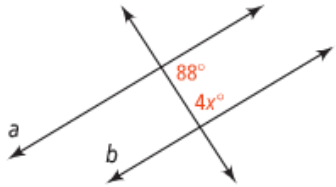


Find the value of x for which $a \parallel b$.

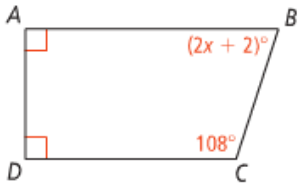
17.



18.



19. What is the value of x?



Do you UNDERSTAND?

20. **Reasoning** Can a pair of lines be both parallel and skew? Explain.
21. **Open-Ended** Give an example of parallel lines in the real world. Then describe how you could prove that the lines are parallel.
22. **Reasoning** Lines ℓ, r, and s are coplanar. Suppose ℓ is perpendicular to r and r is perpendicular to s. Is ℓ perpendicular to s? Explain.

3-6 Constructing Parallel and Perpendicular Lines

Objective To construct parallel and perpendicular lines

Check out the angles formed by the folded lines.

Getting Ready!

Draw a line m on a sheet of paper. Fold your paper so that line m falls on itself. Label your fold line n. Fold your paper again so that n falls on itself. Label your new fold line p. How are m and p related? How do you know?

In the Solve It, you used paper-folding to construct lines.

Dynamic Activity Constructing Parallel and Perpendicular Lines

Essential Understanding You can also use a straightedge and a compass to construct parallel and perpendicular lines.

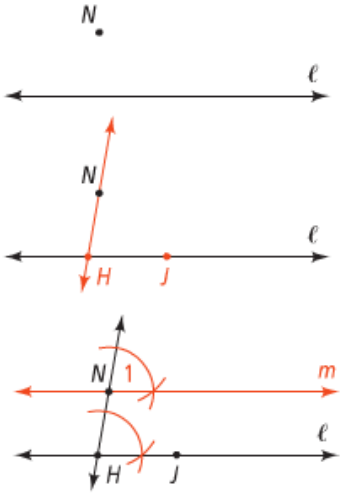
In Lesson 3-5, you learned that through a point not on a line, there is a unique line parallel to the given line. Problem 1 shows the construction of this line.

Problem 1 Constructing Parallel Lines

Construct the line parallel to a given line and through a given point that is not on the line.

Given: line ℓ and point N not on ℓ

Construct: line m through N with $m \parallel \ell$



Step 1 Label two points H and J on ℓ. Draw $\overleftrightarrow{HN}$.

Think

What type of angles are $\angle 1$ and $\angle NHJ$?
They are corresponding angles.

Step 2 At N, construct $\angle 1$ congruent to $\angle NHJ$. Label the new line m.

$m \parallel \ell$

Got It? 1. **Reasoning** Why must lines ℓ and m be parallel?

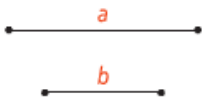
Problem 2 Constructing a Special Quadrilateral

Construct a quadrilateral with one pair of parallel sides of lengths a and b.

Plan

How do you know which constructions to use?
Try sketching the final figure. This can help you visualize the construction steps you will need.

Given: segments of lengths a and b



Construct: quadrilateral $ABYZ$ with
$AZ = a$, $BY = b$, and $\overleftrightarrow{AZ} \parallel \overleftrightarrow{BY}$

Think

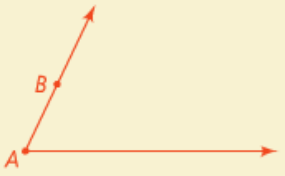
You need a pair of parallel sides, so construct parallel lines as you did in Problem 1. Start by drawing a ray with endpoint A. Then draw $\overrightarrow{AB}$ such that point B is not on the first ray.

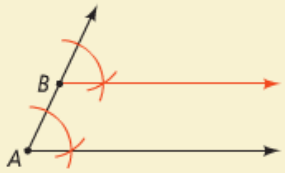
Construct congruent corresponding angles to finish your parallel lines.

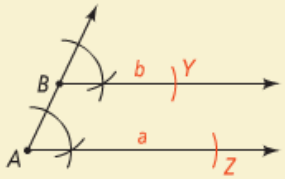
Now you need sides of lengths a and b. In Lesson 1-6, you learned how to construct congruent segments. Construct Y and Z so that $BY = b$ and $AZ = a$.

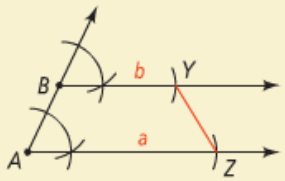
Draw $\overline{YZ}$.

Write









$ABYZ$ is a quadrilateral with parallel sides of lengths a and b.

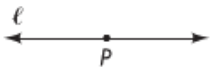
Got It? **2. a.** Draw a segment. Label its length m. Construct quadrilateral $ABCD$ with $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$, so that $AB = m$ and $CD = 2m$.

b. Reasoning Suppose you and a friend both use the steps in Problem 2 to construct $ABYZ$ independently. Will your quadrilaterals necessarily have the same angle measures and side lengths? Explain.

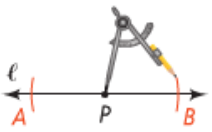
Problem 3 Perpendicular at a Point on a Line

Construct the perpendicular to a given line at a given point on the line.

Given: point P on line ℓ
Construct: $\overleftrightarrow{CP}$ with $\overleftrightarrow{CP} \perp \ell$



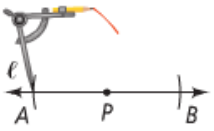
Step 1 Construct two points on ℓ that are equidistant from P. Label the points A and B.



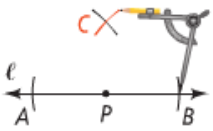
Think

Why is it important to open your compass wider?
If you don't, you won't be able to draw intersecting arcs above point P.

Step 2 Open the compass wider so the opening is greater than $\frac{1}{2}AB$. With the compass tip on A, draw an arc above point P.

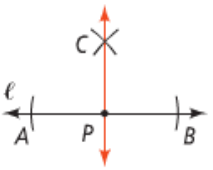


Step 3 Without changing the compass setting, place the compass point on point B. Draw an arc that intersects the arc from Step 2. Label the point of intersection C.



Step 4 Draw $\overleftrightarrow{CP}$.

$\overleftrightarrow{CP} \perp \ell$



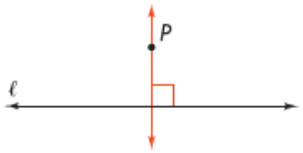
Got It? **3.** Use a straightedge to draw $\overleftrightarrow{EF}$. Construct $\overleftrightarrow{FG}$ so that $\overleftrightarrow{FG} \perp \overleftrightarrow{EF}$ at point F.

You can also construct a perpendicular line from a point to a line. This perpendicular line is unique according to the Perpendicular Postulate. You will prove in Chapter 5 that the shortest path from any point to a line is along this unique perpendicular line.

take note

Postulate 3-4 Perpendicular Postulate

Through a point not on a line, there is one and only one line perpendicular to the given line.



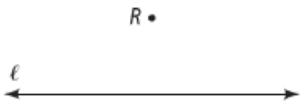
There is exactly one line through P perpendicular to ℓ.

Problem 4 Perpendicular From a Point to a Line

Construct the perpendicular to a given line through a given point not on the line.

Given: line ℓ and point R not on ℓ

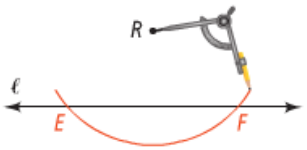
Construct: $\overleftrightarrow{RG}$ with $\overleftrightarrow{RG} \perp \ell$



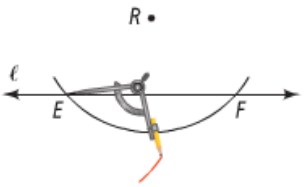
Plan

Can you use similar steps for this problem as in Problem 3?
Yes. Mark two points on ℓ that are equidistant from the given point. Then draw two intersecting arcs.

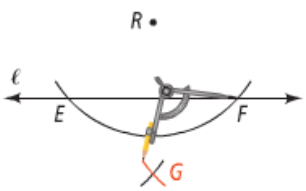
Step 1 Open your compass to a size greater than the distance from R to ℓ. With the compass on point R, draw an arc that intersects ℓ at two points. Label the points E and F.



Step 2 Place the compass point on E and make an arc.

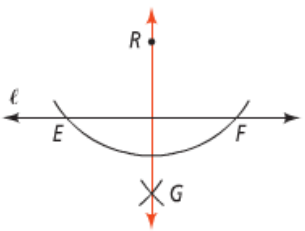


Step 3 Keep the same compass setting. With the compass tip on F, draw an arc that intersects the arc from Step 2. Label the point of intersection G.



Step 4 Draw $\overleftrightarrow{RG}$.

$\overleftrightarrow{RG} \perp \ell$



Got It? **4.** Draw $\overleftrightarrow{CX}$ and a point Z not on $\overleftrightarrow{CX}$. Construct $\overleftrightarrow{ZB}$ so that $\overleftrightarrow{ZB} \perp \overleftrightarrow{CX}$.

Lesson Check

Do you know HOW?

1. Draw a line ℓ and a point P not on the line. Construct the line through P parallel to line ℓ.
2. Draw $\overleftrightarrow{QR}$ and a point S on the line. Construct the line perpendicular to $\overleftrightarrow{QR}$ at point S.
3. Draw a line w and a point X not on the line. Construct the line perpendicular to line w at point X.

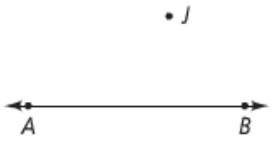
Do you UNDERSTAND?

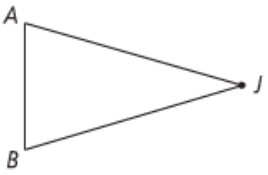
4. In Problem 3, is $\overline{AC}$ congruent to $\overline{BC}$? Explain.
5. Suppose you use a wider compass setting in Step 1 of Problem 4. Will you construct a different perpendicular line? Explain.
6. **Compare and Contrast** How are the constructions in Problems 3 and 4 similar? How are they different?

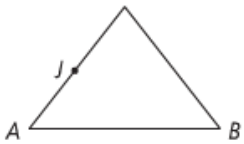
Practice and Problem-Solving Exercises

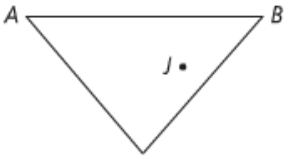
A Practice

For Exercises 7–10, draw a figure like the given one. Then construct the line through point J that is parallel to $\overleftrightarrow{AB}$. See Problem 1.

7. 

8. 

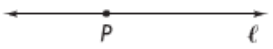
9. 

10. 

For Exercises 11–13, draw two segments. Label their lengths a and b. Construct a quadrilateral with one pair of parallel sides as described. See Problem 2.

11. The sides have length a and b.
12. The sides have length $2a$ and b.
13. The sides have length a and $\frac{1}{2}b$.

For Exercises 14 and 15, draw a figure like the given one. Then construct the line that is perpendicular to ℓ at point P. See Problem 3.

14. 

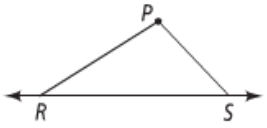
15.

For Exercises 16–18, draw a figure like the given one. Then construct the line through point P that is perpendicular to $\overleftrightarrow{RS}$.

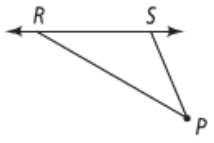
See Problem 4.

16. P •

17.



18.

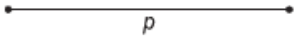


B Apply

19. Think About a Plan Draw an acute angle. Construct an angle congruent to your angle so that the two angles are alternate interior angles.

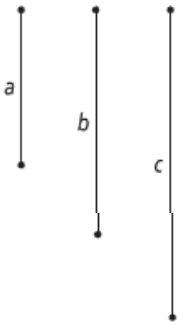
- What does a sketch of the angle look like?
- Which construction(s) should you use?

20. Constructions Construct a square with side length p.



21. Writing Explain how to use the Converse of the Alternate Interior Angles Theorem to construct a line parallel to the given line through a point not on the line. (*Hint:* See Exercise 19.)

For Exercises 22–28, use the segments at the right.



22. Draw a line m. Construct a segment of length b that is perpendicular to line m.

23. Construct a rectangle with base b and height c.

24. Construct a square with sides of length a.

25. Construct a rectangle with one side of length a and a diagonal of length b.

26. a. Construct a quadrilateral with a pair of parallel sides of length c.

b. Make a Conjecture What appears to be true about the other pair of sides in the quadrilateral you constructed?

c. Use a protractor, a ruler, or both to check the conjecture you made in part (b).

27. Construct a right triangle with legs of lengths a and b.

28. a. Construct a triangle with sides of lengths a, b, and c.

b. Construct the midpoint of each side of the triangle.

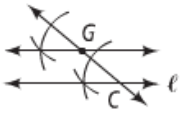
c. Form a new triangle by connecting the midpoints.

d. Make a Conjecture How do the sides of the smaller triangle and the sides of the larger triangle appear to be related?

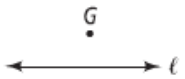
e. Use a protractor, ruler, or both to check the conjecture you made in part (d).

29. Constructions The diagrams below show steps for a parallel line construction.

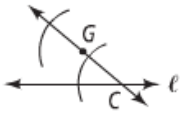
I.



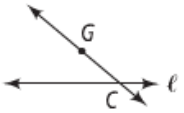
II.



III.



IV.



a. List the construction steps in the correct order.

b. For the steps that use a compass, describe the location(s) of the compass point.

Challenge Draw $\overline{DG}$. Construct a quadrilateral with diagonals that are congruent to $\overline{DG}$, bisect each other, and meet the given conditions. Describe the figure.

30. The diagonals are not perpendicular.

31. The diagonals are perpendicular.

Construct a rectangle with side lengths a and b that meets the given condition.

32. $b = 2a$ **33.** $b = \frac{1}{2}a$ **34.** $b = \frac{1}{3}a$ **35.** $b = \frac{2}{3}a$

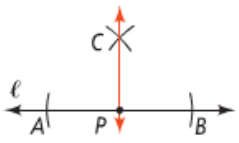
Construct a triangle with side lengths a, b, and c that meets the given conditions. If such a triangle is not possible, explain.

36. $a = b = c$ **37.** $a = b = 2c$ **38.** $a = 2b = 2c$ **39.** $a = b + c$

Standardized Test Prep

SAT/ACT

40. The diagram at the right shows the construction of $\overleftrightarrow{CP}$ perpendicular to line ℓ through point P. Which of the following *must* be true?



(A) $\overleftrightarrow{CB} \parallel \overleftrightarrow{AB}$ (C) $\overleftrightarrow{AC} \parallel \overleftrightarrow{CB}$

(B) $CP = \frac{1}{2}AB$ (D) $\overline{AC} \cong \overline{BC}$

41. Suppose you construct lines ℓ, m, and n so that $\ell \perp m$ and $\ell \parallel n$. Which of the following is true?

(F) $m \parallel n$ (G) $m \parallel \ell$ (H) $n \perp \ell$ (I) $n \perp m$

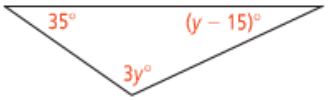
Short Response

42. For any two points, you can draw one segment. For any three noncollinear points, you can draw three segments. For any four noncollinear points, you can draw six segments. How many segments can you draw for eight noncollinear points? Explain your reasoning.

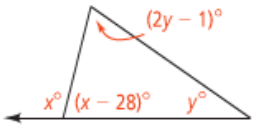
Mixed Review

Find each missing angle measure. See Lesson 3-5.

43.



44.



Get Ready! **To prepare for Lesson 3-7, do Exercises 45–47.**

Simplify each ratio. See p. 831.

45. $\frac{2 - (-3)}{6 - (-4)}$ **46.** $\frac{1 - 4}{-2 - 1}$ **47.** $\frac{12 - 6}{2 - 5}$

3-7 Equations of Lines in the Coordinate Plane

Objective To graph and write linear equations

Think back! What did you learn in algebra that relates to steepness?

SOLVE IT!

Getting Ready!

Ski resorts often use steepness to rate the difficulty of their hills. The steeper the hill, the higher the difficulty rating. Below are sketches of three new hills at a particular resort. Use each rating level only once. Which hill gets which rating? Explain.

Difficulty Ratings
- Easiest
- Intermediate
- Difficult

Lesson Vocabulary
- slope
- slope-intercept form
- point-slope form

The Solve It involves using vertical and horizontal distances to determine steepness. The steepest hill has the greatest *slope*. In this lesson you will explore the concept of slope and how it relates to both the graph and the equation of a line.

Essential Understanding You can graph a line and write its equation when you know certain facts about the line, such as its slope and a point on the line.

take note

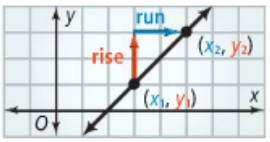
Key Concept Slope

Definition
The **slope** m of a line is the ratio of the vertical change (rise) to the horizontal change (run) between any two points.

Symbols
A line contains the points (x_1, y_1) and (x_2, y_2).

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

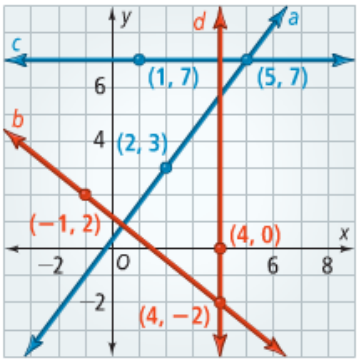
Diagram



Problem 1 Finding Slopes of Lines

Plan

How do you know which numbers go where in a formula? For slope, you can choose either point's y-coordinate as y_2. Just be sure to use the same point's x-coordinate as x_2.



A What is the slope of line b?

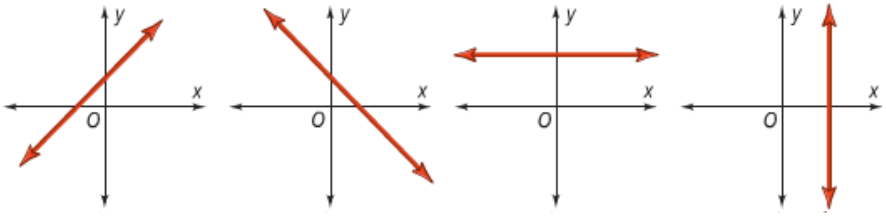
$$m = \frac{2 - (-2)}{-1 - 4}$$
$$= \frac{4}{-5}$$
$$= -\frac{4}{5}$$

B What is the slope of line d?

$$m = \frac{0 - (-2)}{4 - 4}$$
$$= \frac{2}{0}$$ Undefined

Got It? 1. Use the graph in Problem 1.
- **a.** What is the slope of line a?
- **b.** What is the slope of line c?

As you saw in Problem 1 and Got It 1 the slope of a line can be positive, negative, zero, or undefined. The sign of the slope tells you whether the line rises or falls to the right. A slope of zero tells you that the line is horizontal. An undefined slope tells you that the line is vertical.



You can graph a line when you know its equation. The equation of a line has different forms. Two forms are shown below. Recall that the y-intercept of a line is the y-coordinate of the point where the line crosses the y-axis.

take note

Key Concept Forms of Linear Equations

Definition	Symbols
The **slope-intercept form** of an equation of a nonvertical line is $y = mx + b$, where m is the slope and b is the y-intercept.	$y = mx + b$ (slope, y-intercept)
The **point-slope form** of an equation of a nonvertical line is $y - y_1 = m(x - x_1)$, where m is the slope and (x_1, y_1) is a point on the line.	$y - y_1 = m(x - x_1)$ (y-coordinate, slope, x-coordinate)

Problem 2 Graphing Lines

Plan

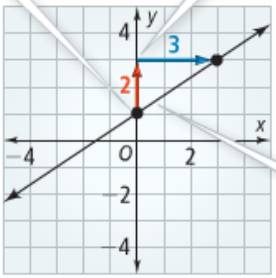
What do you do first?
Determine which form of linear equation you have. Then use the equation to identify the slope and a starting point.

A **What is the graph of $y = \frac{2}{3}x + 1$?**

The equation is in slope-intercept form, $y = mx + b$. The slope m is $\frac{2}{3}$ and the y-intercept b is 1.

Step 1 Graph a point at (0,1).

Step 2 Use the slope $\frac{2}{3}$. Go up 2 units and right 3 units. Graph a point.



Step 3 Draw a line through the two points.

Think

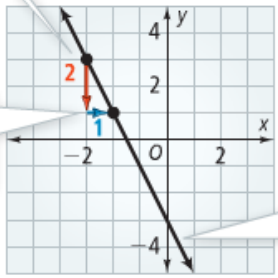
What do you do when the slope is not a fraction?
A rise always needs a run. So, write the integer as a fraction with 1 as the denominator.

B **What is the graph of $y - 3 = -2(x + 3)$?**

The equation is in point-slope form, $y - y_1 = m(x - x_1)$. The slope m is -2 and a point (x_1, y_1) on the line is $(-3, 3)$.

Step 1 Graph a point at $(-3, 3)$.

Step 2 Use the slope -2. Go down 2 units and right 1 unit. Graph a point.



Step 3 Draw a line through the two points.

Got It? **2. a.** Graph $y = 3x - 4$.
b. Graph $y - 2 = -\frac{1}{3}(x - 4)$.

You can write an equation of a line when you know its slope and at least one point on the line.

Problem 3 Writing Equations of Lines

Plan

Which linear equation form should you use?
When you know the slope and the *y*-intercept, use slope-intercept form. When you know the slope and a point on the line, use point-slope form.

A **What is an equation of the line with slope 3 and *y*-intercept −5?**

$y = mx + b$

$m = 3$ $b = -5$

$y = 3x + (-5)$ Substitute 3 for m and -5 for b.

$y = 3x - 5$ Simplify.

B **What is an equation of the line through (−1, 5) with slope 2?**

$y - y_1 = m(x - x_1)$

$y_1 = 5$ $m = 2$ $x_1 = -1$

$y - 5 = 2[x - (-1)]$ Substitute $(-1, 5)$ for (x_1, y_1) and 2 for m.

$y - 5 = 2(x + 1)$ Simplify.

Got It? **3. a.** What is an equation of the line with slope $-\frac{1}{2}$ and *y*-intercept 2?
b. What is an equation of the line through $(-1, 4)$ with slope -3?

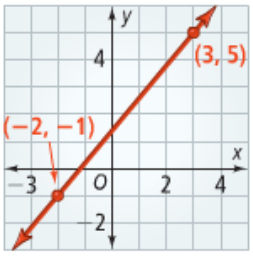
Postulate 1-1 states that through any two points, there is exactly one line. So, you need only two points to write the equation of a line.

Problem 4 Using Two Points to Write an Equation

Plan

What is the first thing you need to know?
It doesn't matter yet what form of linear equation you plan to use. You'll need the slope for *both* slope-intercept form and point-slope form.

What is an equation of the line at the right?



Think

Start by finding the slope m of the line through the given points.

Write

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - (-1)}{3 - (-2)} = \frac{6}{5}$

You have the slope and you know two points on the line. Use point-slope form.

$y - y_1 = m(x - x_1)$

Use either point for (x_1, y_1). For example, you can use (3, 5).

$y - 5 = \frac{6}{5}(x - 3)$

Got It? **4. a.** What is the equation of the line in Problem 4 if you use $(-2, -1)$ instead of $(3, 5)$ in the last step?

b. Rewrite the equations in Problem 4 and part (a) in slope-intercept form and compare them. What can you conclude?

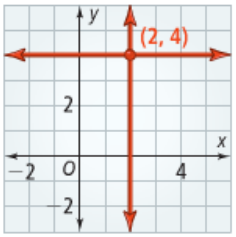
You know that the slope of a horizontal line is 0 and the slope of a vertical line is undefined. Thus, horizontal and vertical lines have easily recognized equations.

Problem 5 Writing Equations of Horizontal and Vertical Lines

Think

How is this different from writing other linear equations?
You don't need the slope. Just locate the point where the line crosses the x-axis (for vertical) or y-axis (for horizontal).

What are the equations for the horizontal and vertical lines through $(2, 4)$?



Every point on the horizontal line through $(2, 4)$ has a y-coordinate of 4. The equation of the line is $y = 4$. It crosses the y-axis at $(0, 4)$.

Every point on the vertical line through $(2, 4)$ has an x-coordinate of 2. The equation of the line is $x = 2$. It crosses the x-axis at $(2, 0)$.

Got It? **5. a.** What are the equations for the horizontal and vertical lines through $(4, -3)$?

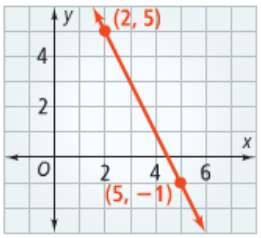
b. Reasoning Can you write the equation of a vertical line in slope-intercept form? Explain.

Lesson Check

Do you know HOW?

For Exercises 1 and 2, find the slope of the line passing through the given points.

1. $(4, 5)$ and $(6, 15)$

2. 

3. What is an equation of a line with slope 8 and y-intercept 10?

4. What is an equation of a line passing through $(3, 3)$ and $(4, 7)$?

Do you UNDERSTAND?

5. **Vocabulary** Explain why you think *slope-intercept form* makes sense as a name for $y = mx + b$. Explain why you think *point-slope form* make sense as a name for $y - y_1 = m(x - x_1)$.

6. **Compare and Contrast** Graph $y = 2x + 5$ and $y = \frac{1}{3}x + 5$. Describe how these lines are alike and how they are different.

7. **Error Analysis** A classmate found the slope of the line passing through $(8, -2)$ and $(8, 10)$, as shown at the right. Describe your classmate's error. Then find the correct slope of the line passing through the given points.

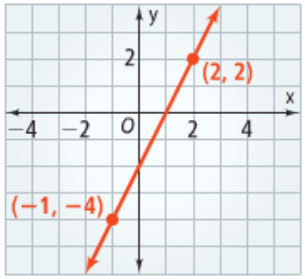
$$m = \frac{8 - 8}{10 - (-2)}$$
$$m = \frac{0}{12}$$
$$m = 0$$

Practice and Problem-Solving Exercises

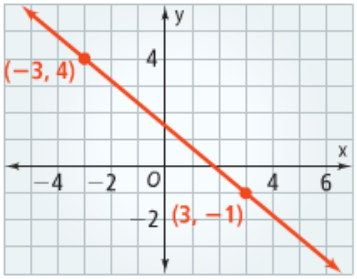
A Practice

Find the slope of the line passing through the given points. See Problem 1.

8.



9.



10. $(4, -6), (7, 2)$

11. $(-3, 7), (-1, 4)$

12. $(-8, 3), (-11, 4)$

13. $(-6, 2), (-7, 10)$

14. $(3, 2), (-6, 2)$

15. $(5, 9), (5, -6)$

Graph each line. See Problem 2.

16. $y = x + 2$

17. $y = 3x + 4$

18. $y = \frac{1}{2}x - 1$

19. $y = -\frac{5}{3}x + 2$

20. $y - 3 = \frac{1}{3}(x - 3)$

21. $y - 1 = -3(x + 2)$

22. $y + 4 = (x - 5)$

23. $y + 1 = -\frac{2}{3}(x + 4)$

Use the given information to write an equation of each line. See Problems 3 and 4.

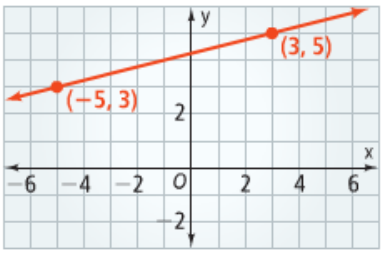
24. slope 3, y-intercept 6

25. slope $\frac{1}{2}$, y-intercept -5

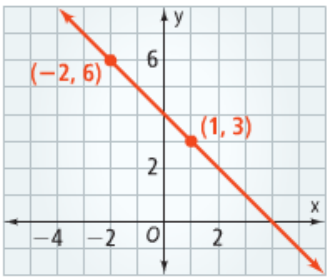
26. slope $\frac{2}{3}$, passes through $(-2, -6)$

27. slope -3, passes through $(4, -1)$

28.



29.



30. passes through $(0, 5)$ and $(5, 8)$

31. passes through $(6, 2)$ and $(2, 4)$

32. passes through $(-4, 4)$ and $(2, 10)$

33. passes through $(-1, 0)$ and $(-3, -1)$

Write the equation of the horizontal and vertical lines though the given point. See Problem 5.

34. $(4, 7)$

35. $(3, -2)$

36. $(0, -1)$

37. $(6, 4)$

B Apply

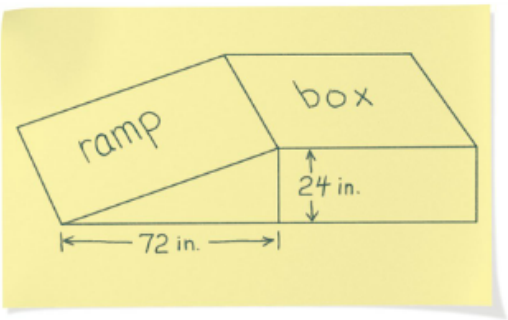
Graph each line.

38. $x = 3$ **39.** $y = -2$ **40.** $x = 9$ **41.** $y = 4$

42. Open-Ended Write equations for three lines that contain the point $(5, 6)$.

43. Think About a Plan You want to construct a "funbox" at a local skate park. The skate park's safety regulations allow for the ramp on the funbox to have a maximum slope of $\frac{4}{11}$. If you use the funbox plan at the right, can you build the ramp to meet the safety regulations? Explain.

- What information do you have that you can use to find the slope?
- How can you compare slopes?



Write each equation in slope-intercept form.

44. $y - 5 = 2(x + 2)$ **45.** $y + 2 = -(x - 4)$ **46.** $-5x + y = 2$ **47.** $3x + 2y = 10$

48. Science The equation $P = \frac{1}{33}d + 1$ represents the pressure P in atmospheres a scuba diver feels d feet below the surface of the water.

a. What is the slope of the line?
b. What does the slope represent in this situation?
c. What is the y-intercept (P-intercept)?
d. What does the y-intercept represent in this situation?

Graph each pair of lines. Then find their point of intersection.

49. $y = -4, x = 6$ **50.** $x = 0, y = 0$ **51.** $x = -1, y = 3$ **52.** $y = 5, x = 4$

53. Accessibility By law, the maximum slope of an access ramp in new construction is $\frac{1}{12}$. The plan for the new library shows a 3-ft height from the ground to the main entrance. The distance from the sidewalk to the building is 10 ft. If you assume the ramp does not have any turns, can you design a ramp that complies with the law? Explain.

54. a. What is the slope of the x-axis? Explain.
b. Write an equation for the x-axis.

55. a. What is the slope of the y-axis? Explain.
b. Write an equation for the y-axis.

56. Reasoning The x-intercept of a line is 2 and the y-intercept is 4. Use this information to write an equation for the line.

57. Coordinate Geometry The vertices of a triangle are $A(0, 0)$, $B(2, 5)$, and $C(4, 0)$.

a. Write an equation for the line through A and B.
b. Write an equation for the line through B and C.
c. Compare the slopes and the y-intercepts of the two lines.

Challenge Do the three points lie on one line? Justify your answer.

58. $(5, 6), (3, 2), (6, 8)$ **59.** $(-2, -2), (4, -4), (0, 0)$ **60.** $(5, -4), (2, 3), (-1, 10)$

Find the value of a such that the graph of the equation has the given slope.

61. $y = \frac{2}{9}ax + 6; m = 2$ **62.** $y = -3ax - 4; m = \frac{1}{2}$ **63.** $y = -4ax - 10; m = -\frac{2}{3}$

Standardized Test Prep

SAT/ACT

64. $\overline{AB}$ has endpoints $A(8, k)$ and $B(7, -3)$. The slope of $\overline{AB}$ is 5. What is k?

(A) 1 (B) 2 (C) 5 (D) 8

65. Two angles of a triangle measure 68 and 54. What is the measure of the third angle?

(F) 14 (G) 58 (H) 122 (I) 180

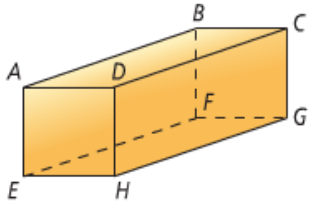
66. Which of the following CANNOT be true?

(A) plane $ABCD \parallel$ plane $EFGH$

(B) Planes $ABCD$ and $CDHG$ intersect in $\overleftrightarrow{CD}$.

(C) $ABCD$ and ABC represent the same plane.

(D) plane $ADHE \parallel$ plane DCG



67. The length of a rectangle is $(x - 2)$ inches and the width is $5x$ inches. Which expression represents the perimeter of the rectangle in inches?

(F) $6x - 2$ (G) $12x - 4$ (H) $5x^2 - 10x$ (I) $10x^2 - 20x$

Short Response

68. One of the angles in a certain linear pair is acute. Your friend says the other angle must be obtuse. Is your friend's conjecture reasonable? Explain.

Mixed Review

For Exercises 69 and 70, construct the geometric figure. See Lesson 3-6.

69. a rectangle with a length twice its width **70.** a square

Name the property that justifies each statement. See Lesson 2-5.

71. $4(2a - 3) = 8a - 12$ **72.** If $b + c = 7$ and $b = 2$, then $2 + c = 7$.

73. $\overline{RS} \cong \overline{RS}$ **74.** If $\angle 1 \cong \angle 4$, then $\angle 4 \cong \angle 1$.

Get Ready! **To prepare for Lesson 3-8, do Exercises 75–77.**

Find the slope of the line passing through the given points. See Lesson 3-7.

75. $(2, 5), (-2, 3)$ **76.** $(0, -5), (2, 0)$ **77.** $(1, 1), (2, -4)$

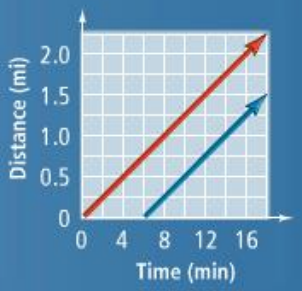
3-8 Slopes of Parallel and Perpendicular Lines

Objective To relate slope to parallel and perpendicular lines

SOLVE IT!

Getting Ready!

You and a friend enjoy exercising together. One day, you are about to go running when your friend receives a phone call. You decide to start running and tell your friend to catch up after the call. The red line represents you and the blue line represents your friend. Will your friend catch up? Explain.



In the Solve It, slope represents the running rate, or speed. According to the graph, you and your friend run at the same speed, so the slopes of the lines are the same. In this lesson, you will learn how to use slopes to determine how two lines relate graphically to each other.

Essential Understanding You can determine whether two lines are parallel or perpendicular by comparing their slopes.

When two lines are parallel, their slopes are the same.

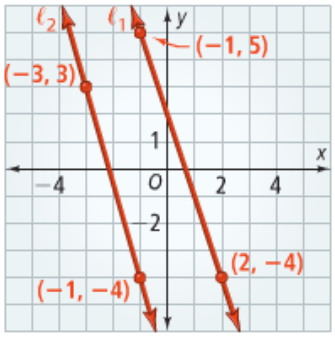
take note

Key Concept Slopes of Parallel Lines

- If two nonvertical lines are parallel, then their slopes are equal.
- If the slopes of two distinct nonvertical lines are equal, then the lines are parallel.
- Any two vertical lines or horizontal lines are parallel.

Problem 1 Checking for Parallel Lines

Are lines ℓ_1 and ℓ_2 parallel? Explain.



Think

Can you tell from the diagram whether the lines are parallel?
No. The lines may look parallel, but you only see a small portion of their graphs. Compare their slopes to know for sure.

Step 1 Find the slope of each line.

$$\text{slope of } \ell_1 = \frac{5 - (-4)}{-1 - 2} = \frac{9}{-3} = -3$$

$$\text{slope of } \ell_2 = \frac{3 - (-4)}{-3 - (-1)} = \frac{7}{-2} = -\frac{7}{2}$$

Step 2 Compare the slopes.

Since $-3 \neq -\frac{7}{2}$, ℓ_1 and ℓ_2 are not parallel.

Got It? **1.** Line ℓ_3 contains $A(-13, 6)$ and $B(-1, 2)$. Line ℓ_4 contains $C(3, 6)$ and $D(6, 7)$. Are ℓ_3 and ℓ_4 parallel? Explain.

Problem 2 Writing Equations of Parallel Lines

What is an equation of the line parallel to $y = -3x - 5$ that contains $(-1, 8)$?

Plan

How does the given line help you?
Parallel lines have the same slope. Once you know the slope of the given line, you know the slope you need to write an equation.

Think	Write
Identify the slope of the given line.	$y = -3x - 5$
You now know the slope of the new line and that it passes through $(-1, 8)$. Use point-slope form to	$y - y_1 = m(x - x_1)$

Got It? **2.** What is an equation of the line parallel to $y = -x - 7$ that contains $(-5, 3)$?

When two lines are perpendicular, the product of their slopes is -1. Numbers with product -1 are opposite reciprocals.

take note

Key Concept Slopes of Perpendicular Lines

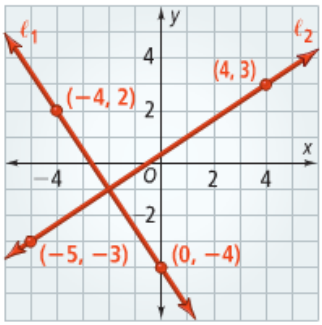
- If two nonvertical lines are perpendicular, then the product of their slopes is -1.
- If the slopes of two lines have a product of -1, then the lines are perpendicular.
- Any horizontal line and vertical line are perpendicular.

Problem 3 Checking for Perpendicular Lines

Plan

Can you tell from the diagram whether the lines are perpendicular?
No. You can tell that the lines intersect, but not necessarily at right angles. So, you need to compare their slopes.

Lines ℓ_1 and ℓ_2 are neither horizontal nor vertical. Are they perpendicular? Explain.



Step 1 Find the slope of each line.

m_1 = slope of ℓ_1 $= \frac{2 - (-4)}{-4 - 0} = \frac{6}{-4} = -\frac{3}{2}$

m_2 = slope of ℓ_2 $= \frac{3 - (-3)}{4 - (-5)} = \frac{6}{9} = \frac{2}{3}$

Step 2 Find the product of the slopes.

$m_1 \cdot m_2 = -\frac{3}{2} \cdot \frac{2}{3} = -1$

Lines ℓ_1 and ℓ_2 are perpendicular because the product of their slopes is -1.

Got It? **3.** Line ℓ_3 contains $A(2, 7)$ and $B(3, -1)$. Line ℓ_4 contains $C(-2, 6)$ and $D(8, 7)$. Are ℓ_3 and ℓ_4 perpendicular? Explain.

Problem 4 Writing Equations of Perpendicular Lines

Think

How is this similar to writing equations of parallel lines?
You follow the same process. The only difference here is that the slopes of perpendicular lines have product -1.

What is an equation of the line perpendicular to $y = \frac{1}{5}x + 2$ that contains $(15, -4)$?

Step 1 Identify the slope of the given line.

$y = \frac{1}{5}x + 2$
↑
slope

Step 2 Find the slope of the line perpendicular to the given line.

$m_1 \cdot m_2 = -1$ The product of the slopes of $\perp$ lines is -1.

$\frac{1}{5} \cdot m_2 = -1$ Substitute $\frac{1}{5}$ for m_1.

$m_2 = -5$ Multiply each side by 5.

Step 3 Use point-slope form to write an equation of the new line.

$y - y_1 = m(x - x_1)$

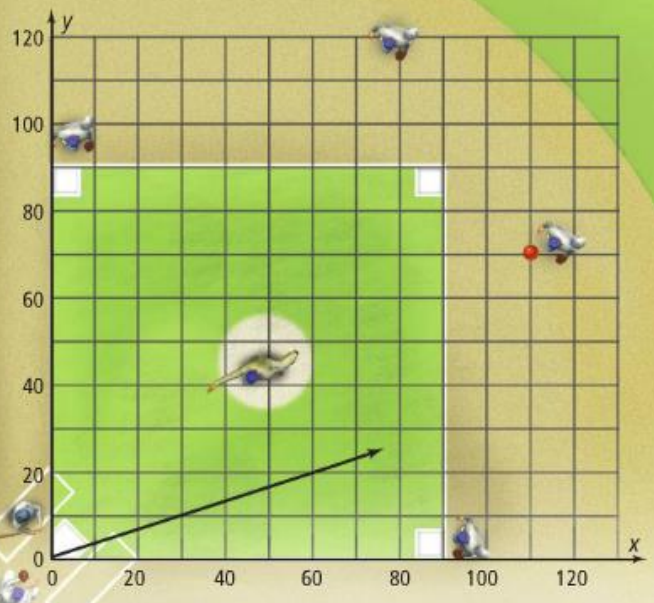
$y - (-4) = -5(x - 15)$ Substitute -5 for m and $(15, -4)$ for (x_1, y_1).

$y + 4 = -5(x - 15)$ Simplify.

Got It? **4.** What is an equation of the line perpendicular to $y = -3x - 5$ that contains $(-3, 7)$?

Problem 5 Writing Equations of Lines

Sports The baseball field below is on a coordinate grid with home plate at the origin. A batter hits a ground ball along the line shown. The player at (110, 70) runs along a path perpendicular to the path of the baseball. What is an equation of the line on which the player runs?



Step 1 Find the slope of the baseball's path.

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{20 - 10}{60 - 30} = \frac{10}{30} = \frac{1}{3}$$

Points (30, 10) and (60, 20) are on the baseball's path.

Step 2 Find the slope of a line perpendicular to the baseball's path.

$m_1 \cdot m_2 = -1$ The product of the slopes of $\perp$ lines is -1.

$\frac{1}{3} \cdot m_2 = -1$ Substitute $\frac{1}{3}$ for m_1.

$m_2 = -3$ Multiply each side by 3.

Think

Which linear equation form should you use?
You know the slope. The player is located at a point on the line. Use point-slope form.

Step 3 Write an equation of the line on which the player runs.
The slope is -3 and a point on the line is (110, 70).

$y - y_1 = m(x - x_1)$ Point-slope form

$y - 70 = -3(x - 110)$ Substitute -3 for m and (110, 70) for (x_1, y_1).

Got It? **5.** Suppose a second player standing at (90, 40) misses the ball, turns around, and runs on a path parallel to the baseball's path. What is an equation of the line representing this player's path?

Lesson Check

Do you know HOW?

$\overleftrightarrow{AB}$ contains points A and B. $\overleftrightarrow{CD}$ contains points C and D. Are $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ *parallel, perpendicular,* or *neither*? Explain.

1. $A(-8, 3), B(-4, 11), C(-1, 3), D(1, 2)$
2. $A(3, 5), B(2, -1), C(7, -2), D(10, 16)$
3. $A(3, 1), B(4, 1), C(5, 9), D(2, 6)$
4. What is an equation of the line perpendicular to $y = -4x + 1$ that contains $(2, -3)$?

Do you UNDERSTAND?

5. **Error Analysis** Your classmate tries to find an equation for a line parallel to $y = 3x - 5$ that contains $(-4, 2)$. What is your classmate's error?

slope of given line $= 3$
slope of parallel line $= \frac{1}{3}$
$y - y_1 = m(x - x_1)$
$y - 2 = \frac{1}{3}(x + 4)$

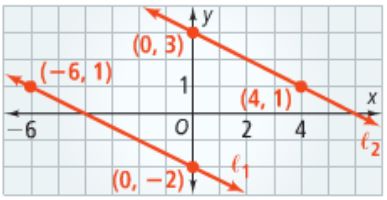
6. **Compare and Contrast** What are the differences between the equations of parallel lines and the equations of perpendicular lines? Explain.

Practice and Problem-Solving Exercises

A Practice

For Exercises 7–10, are lines ℓ_1 and ℓ_2 parallel? Explain. See Problem 1.

7.



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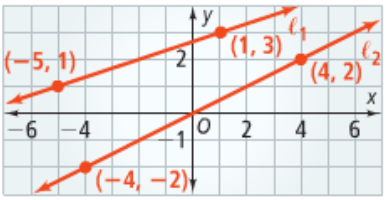

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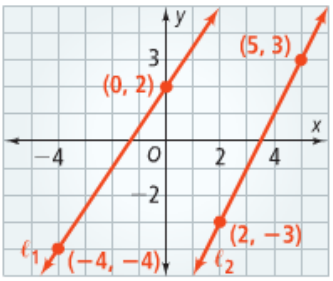

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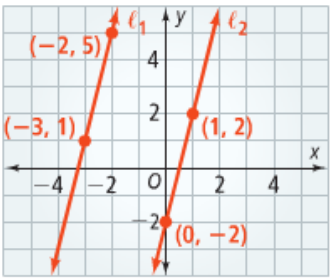
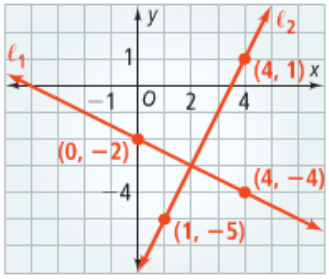


Write an equation of the line parallel to the given line that contains C. See Problem 2.

11. $C(0, 3); y = -2x + 1$
12. $C(6, 0); y = \frac{1}{3}x$
13. $C(-2, 4); y = \frac{1}{2}x + 2$
14. $C(6, -2); y = -\frac{3}{2}x + 6$

For Exercises 15–18, are lines ℓ_1 and ℓ_2 perpendicular? Explain. See Problem 3.

15.



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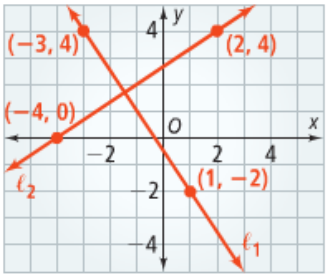

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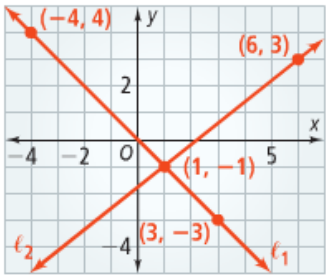

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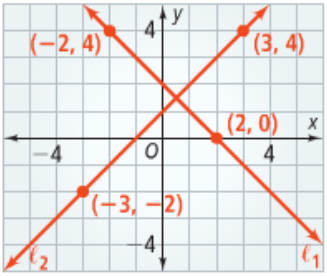


Write an equation of the line perpendicular to the given line that contains P. See Problem 4.

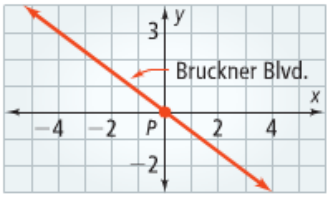
19. $P(6, 6); y = \frac{2}{3}x$

20. $P(4, 0); y = \frac{1}{2}x - 5$

21. $P(4, 4); y = -2x - 8$

22. City Planning City planners want to construct a bike path perpendicular to Bruckner Boulevard at point P. An equation of the Bruckner Boulevard line is $y = -\frac{3}{4}x$. Find an equation of the line for the bike path.

See Problem 5.



B Apply

Rewrite each equation in slope-intercept form, if necessary. Then determine whether the lines are parallel. Explain.

23. $y = -x + 6$
$x + y = 20$

24. $y - 7x = 6$
$y + 7x = 8$

25. $3x + 4y = 12$
$6x + 2y = 6$

26. $2x + 5y = -1$
$10y = -4x - 20$

27. Think About a Plan Line ℓ_1 contains $(-4, 1)$ and $(2, 5)$ and line ℓ_2 contains $(3, 0)$ and $(-3, k)$. What value of k makes ℓ_1 and ℓ_2 parallel?

- For ℓ_1 and ℓ_2 to be parallel, what must be true of their slopes?
- What expressions represent the slopes of ℓ_1 and ℓ_2?

28. Open-Ended Write equations for two perpendicular lines that have the same y-intercept and do not pass through the origin.

29. Writing Can the y-intercepts of two nonvertical parallel lines be the same? Explain.

Use slopes to determine whether the opposite sides of quadrilateral *ABCD* are parallel.

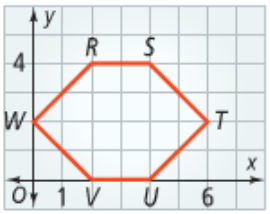
30. $A(0, 2), B(3, 4), C(2, 7), D(-1, 5)$

31. $A(-3, 1), B(1, -2), C(0, -3), D(-4, 0)$

32. $A(1, 1), B(5, 3), C(7, 1), D(3, 0)$

33. $A(1, 0), B(4, 0), C(3, -3), D(-1, -3)$

34. Reasoning Are opposite sides of hexagon *RSTUVW* at the right parallel? Justify your answer.



35. Which line is perpendicular to $3y + 2x = 12$?

(A) $6x - 4y = 24$

(B) $y + 3x = -2$

(C) $2x + 3y = 6$

(D) $y = -2x + 6$

Rewrite each equation in slope-intercept form, if necessary. Then determine whether the lines are perpendicular. Explain.

36. $y = -x - 7$
$y - x = 20$

37. $y = 3$
$x = -2$

38. $2x - 7y = -42$
$4y = -7x - 2$

Developing Proof **Explain why each theorem is true for three lines in the coordinate plane.**

39. Theorem 3-7: If two lines are parallel to the same line, then they are parallel to each other.

40. Theorem 3-8: In a plane, if two lines are perpendicular to the same line, then they are parallel to each other.

41. Rail Trail A community recently converted an old railroad corridor into a recreational trail. The graph at the right shows a map of the trail on a coordinate grid. They plan to construct a path to connect the trail to a parking lot. The new path will be perpendicular to the recreational trail.

a. Write an equation of the line representing the new path.

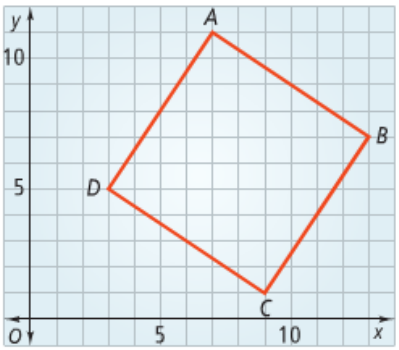
b. What are the coordinates of the point at which the path will meet the recreational trail?

c. If each grid space is 25 yd by 25 yd, how long is the path to the nearest yard?

42. Reasoning Is a triangle with vertices $G(3, 2)$, $H(8, 5)$, and $K(0, 10)$ a right triangle? Justify your answer.

43. Graphing Calculator $\overleftrightarrow{AB}$ contains points $A(-3, 2)$ and $B(5, 1)$. $\overleftrightarrow{CD}$ contains points $C(2, 7)$ and $D(1, -1)$. Use your graphing calculator to find the slope of $\overleftrightarrow{AB}$. Enter the x-coordinates of A and B into the L1 list of your list editor. Enter the y-coordinates into the L2 list. In your stat **CALC** menu select **LinReg ($ax + b$)**. Press enter to find the slope a. Repeat to find the slope of $\overleftrightarrow{CD}$. Are $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ *parallel*, *perpendicular*, or *neither*?

Challenge **For Exercises 44 and 45, use the graph at the right.**



44. Show that the diagonals of the figure are congruent.

45. Show that the diagonals of the figure are perpendicular bisectors of each other.

46. **a.** Graph the points $P(2, 2)$, $Q(7, 4)$, and $R(3, 5)$.
b. Find the coordinates of a point S that, along with points P, Q, and R, will form the vertices of a quadrilateral with opposite sides parallel. Graph the quadrilateral.
c. Repeat part (b) to find a different point S. Graph the new quadrilateral.

47. **Algebra** A triangle has vertices $L(-5, 6)$, $M(-2, -3)$, and $N(4, 5)$. Write an equation for the line perpendicular to $\overline{LM}$ that contains point N.

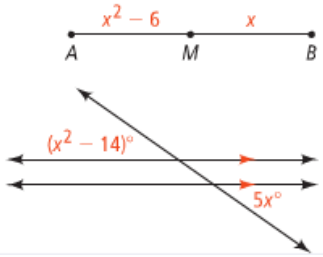
Standardized Test Prep

GRIDDED RESPONSE

SAT/ACT

48. $\triangle ABC$ is right with right angle C. The slope of $\overline{AC}$ is -2. What is the slope of $\overline{BC}$?

49. In the diagram at the right, M is the midpoint of $\overline{AB}$. What is AB?



50. What is the distance between $(-4.5, 1.2)$ and $(3.5, -2.8)$ to the nearest tenth?

51. What is the value of x in the diagram at the right?

52. The perimeter of a square is 20 ft. What is the area of the square in square feet?

Mixed Review

Algebra **Write an equation for the line containing the given points.** See Lesson 3-7.

53. $A(0, 3)$, $B(6, 0)$ **54.** $C(-4, 2)$, $D(-1, 7)$ **55.** $E(3, -2)$, $F(-5, -8)$

Name the property that justifies each statement. See Lesson 2-5.

56. $\angle 4 \cong \angle 4$

57. If $m\angle B = 8$, then $2m\angle B = 16$.

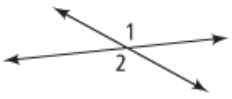
58. $-3x + 6 = 3(-x + 2)$

59. If $\overline{RS} \cong \overline{MN}$, then $\overline{MN} \cong \overline{RS}$.

Get Ready! **To prepare for Lesson 4-1, do Exercises 60–62.**

Are $\angle 1$ and $\angle 2$ congruent? Explain. See Lessons 1-5 and 2-6.

60.



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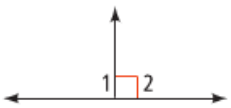
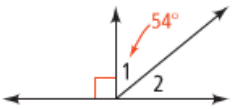
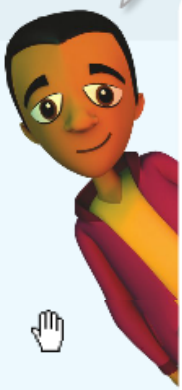


62.



3 *Pull It* All Together

To solve these problems, you will pull together many concepts and skills that you have studied about parallel lines.



BIG idea Reasoning and Proof

You can prove that lines are parallel if you know that certain pairs of angles formed by the lines and a transversal are congruent.

Task 1

You want to put tape on the ground to mark the lines for a volleyball court. What is the most efficient way to make sure that the opposite sides of the court are parallel? Support your answer with a diagram.

BIG idea Reasoning and Proof

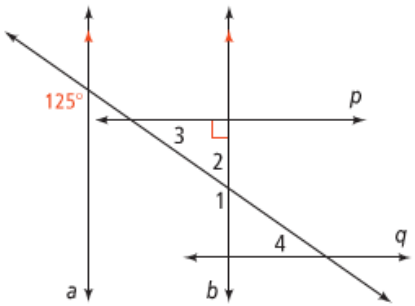
You can prove that lines are parallel if you know that certain pairs of angles formed by the lines and a transversal are congruent.

BIG idea Measurement

You can find missing angle measures in triangles by using the fact that the sum of the measures of the angles of a triangle is 180.

Task 2

In the diagram below, $a \parallel b$. For lines p and q to be parallel, what is $m\angle 4$? Explain.



BIG idea Coordinate Geometry

You can write the equation of a line by using its slope and y-intercept.

Task 3

$\overleftrightarrow{AB}$ contains points $A(-6, -1)$ and $B(1, 4)$. $\overleftrightarrow{CD}$ contains point $D(7, 2)$. If $\angle ABC \cong \angle BCD$ and $m\angle ABC = 90$, what is an equation of $\overleftrightarrow{CD}$? (*Hint:* Sketch a graph.)

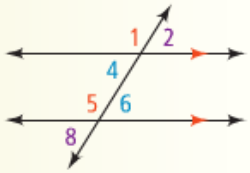
3 Chapter Review

Connecting BIG ideas and Answering the Essential Questions

1 Reasoning and Proof
You can prove that two lines are parallel by using special angle relationships and the relationships of two lines to a third line.

Parallel Lines and Angle Pairs (Lessons 3-2 and 3-3)

$\angle 1 \cong \angle 5$
$\angle 4 \cong \angle 6$
$m\angle 4 + m\angle 5 = 180$
$\angle 2 \cong \angle 8$

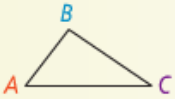


Parallel and Perpendicular Lines (Lesson 3-4)

$a \parallel b$ and $b \parallel c \rightarrow a \parallel c$
$a \perp b$ and $a \perp c \rightarrow b \parallel c$

2 Measurement
The sum of the measures of the angles of a triangle is 180.

Parallel Lines and Triangles (Lesson 3-5)



$m\angle A + m\angle B + m\angle C = 180$

3 Coordinate Geometry
You can write the equation of a line by using its slope and a point on the line.

Lines in the Coordinate Plane (Lesson 3-7)

Slope-intercept form: $y = mx + b$
Point-slope form: $y - y_1 = m(x - x_1)$

Slopes of Parallel and Perpendicular Lines (Lesson 3-8)

Parallel lines: equal slopes
Perpendicular lines: product of slopes is -1

Chapter Vocabulary

- alternate exterior angles (p. 142)
- alternate interior angles (p. 142)
- auxiliary line (p. 172)
- corresponding angles (p. 142)
- exterior angle of a polygon (p. 173)
- flow proof (p. 158)
- parallel lines (p. 140)
- parallel planes (p. 140)
- point-slope form (p. 190)
- remote interior angles (p. 173)
- same-side interior angles (p. 142)
- skew lines (p. 140)
- slope (p. 189)
- slope-intercept form (p. 190)
- transversal (p. 141)

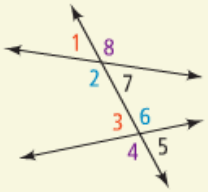
Choose the correct term to complete each sentence.

1. A(n) ___?___ intersects two or more coplanar lines at distinct points.
2. The measure of a(n) ___?___ of a triangle is equal to the sum of the measures of its two remote interior angles.
3. The linear equation $y - 3 = 4(x + 5)$ is in ___?___ form.
4. When two coplanar lines are cut by a transversal, the angles formed between the two lines and on opposite sides of the transversal are ___?___.
5. Noncoplanar lines that do not intersect are ___?___.
6. The linear equation $y = 3x - 5$ is in ___?___ form.

3-1 Lines and Angles

Quick Review

A **transversal** is a line that intersects two or more coplanar lines at distinct points.



∠1 and ∠3 are **corresponding angles.**

∠2 and ∠6 are **alternate interior angles.**

∠2 and ∠3 are **same-side interior angles.**

∠4 and ∠8 are **alternate exterior angles.**

Example

Name two other pairs of corresponding angles in the diagram above.

∠5 and ∠7
∠2 and ∠4

Exercises

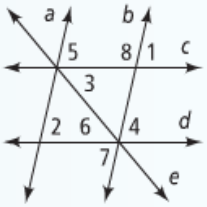
Identify all numbered angle pairs that form the given type of angle pair. Then name the two lines and transversal that form each pair.

7. alternate interior angles

8. same-side interior angles

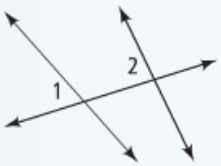
9. corresponding angles

10. alternate exterior angles



Classify the angle pair formed by ∠1 and ∠2.

11.



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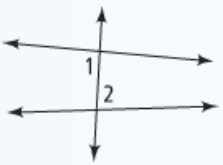


3-2 Properties of Parallel Lines

Quick Review

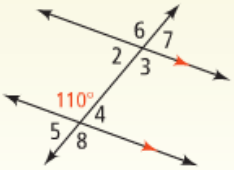
If two parallel lines are cut by a transversal, then

- corresponding angles, alternate interior angles, and alternate exterior angles are congruent
- same-side interior angles are supplementary

Example

Which other angles measure 110?

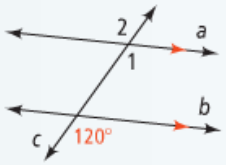
∠6 (corresponding angles)
∠3 (alternate interior angles)
∠8 (vertical angles)



Exercises

Find $m\angle 1$ and $m\angle 2$. Justify your answers.

13.



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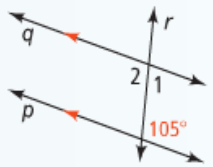
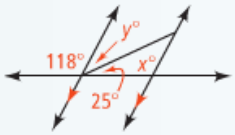


15. Find the values of x and y in the diagram below.



3-3 Proving Lines Parallel

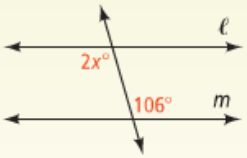
Quick Review

If two lines and a transversal form
- congruent corresponding angles,
- congruent alternate interior angles,
- congruent alternate exterior angles, or
- supplementary same-side interior angles,

then the two lines are parallel.

Example

What is the value of x for which $\ell \parallel m$?



The given angles are alternate interior angles. So, $\ell \parallel m$ if the given angles are congruent.

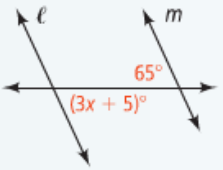
$2x = 106$ Congruent $\measuredangle$s have equal measures.

$x = 53$ Divide each side by 2.

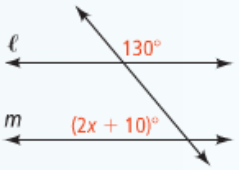
Exercises

Find the value of x for which $\ell \parallel m$.

16.



17.



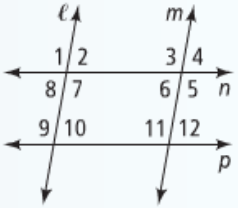
Use the given information to decide which lines, if any, are parallel. Justify your conclusion.

18. $\angle 1 \cong \angle 9$

19. $m\angle 3 + m\angle 6 = 180$

20. $m\angle 2 + m\angle 3 = 180$

21. $\angle 5 \cong \angle 11$



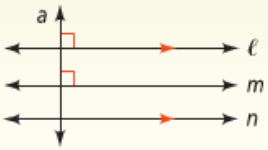
3-4 Parallel and Perpendicular Lines

Quick Review

- Two lines $\parallel$ to the same line are $\parallel$ to each other.
- In a plane, two lines $\perp$ to the same line are $\parallel$.
- In a plane, if one line is $\perp$ to one of two $\parallel$ lines, then it is $\perp$ to both $\parallel$ lines.

Example

What are the pairs of parallel and perpendicular lines in the diagram?

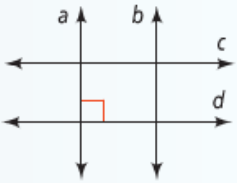


$\ell \parallel n$, $\ell \parallel m$, and $m \parallel n$.

$a \perp \ell$, $a \perp m$, and $a \perp n$.

Exercises

Use the diagram at the right to complete each statement.



22. If $b \perp c$ and $b \perp d$, then c ? d.

23. If $c \parallel d$, then ? $\perp c$.

24. Maps Morris Avenue intersects both 1st Street and 3rd Street at right angles. 3rd Street is parallel to 5th Street. How are 1st Street and 5th Street related? Explain.

3-5 Parallel Lines and Triangles

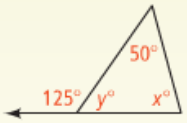
Quick Review

The sum of the measures of the angles of a triangle is 180.

The measure of each **exterior angle** of a triangle equals the sum of the measures of its two **remote interior angles.**

Example

What are the values of x and y?



$x + 50 = 125$ Exterior Angle Theorem

$x = 75$ Simplify.

$x + y + 50 = 180$ Triangle Angle-Sum Theorem

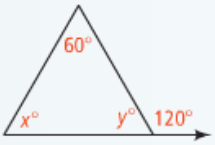
$75 + y + 50 = 180$ Substitute 75 for x.

$y = 55$ Simplify.

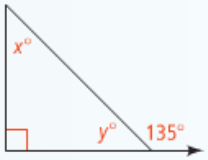
Exercises

Find the values of the variables.

25.



26.



The measures of the three angles of a triangle are given. Find the value of x.

27. $x, 2x, 3x$

28. $x + 10, x - 20, x + 25$

29. $20x + 10, 30x - 2, 7x + 1$

3-6 Constructing Parallel and Perpendicular Lines

Quick Review

You can use a compass and a straightedge to construct

- a line parallel to a given line through a point not on the line
- a line perpendicular to a given line through a point on the line, or through a point not on the line

Example

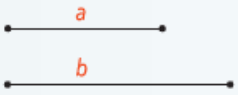
Which step of the parallel lines construction guarantees the lines are parallel?

The parallel lines construction involves constructing a pair of congruent angles. Since the congruent angles are corresponding angles, the lines are parallel.

Exercises

30. Draw a line m and point Q not on m. Construct a line perpendicular to m through Q.

Use the segments below.



31. Construct a rectangle with side lengths a and b.

32. Construct a rectangle with side lengths a and $2b$.

33. Construct a quadrilateral with one pair of parallel opposite sides, each side of length $2a$.

3-7 Equations of Lines in the Coordinate Plane

Quick Review

Slope-intercept form is $y = mx + b$, where m is the slope and b is the y-intercept.

Point-slope form is $y - y_1 = m(x - x_1)$, where m is the slope and (x_1, y_1) is a point on the line.

Example

What is an equation of the line with slope -5 and y-intercept 6?

Use slope-intercept form: $y = -5x + 6$.

Example

What is an equation of the line through $(-2, 8)$ with slope 3?

Use point-slope form: $y - 8 = 3(x + 2)$.

Exercises

Find the slope of the line passing through the points.

34. $(6, -2), (1, 3)$

35. $(-7, 2), (-7, -5)$

36. Name the slope and y-intercept of $y = 2x - 1$. Then graph the line.

37. Name the slope of and a point on $y - 3 = -2(x + 5)$. Then graph the line.

Write an equation of the line.

38. slope $-\frac{1}{2}$, y-intercept 12

39. slope 3, passes through $(1, -9)$

40. passes through $(4, 2)$ and $(3, -2)$

3-8 Slopes of Parallel and Perpendicular Lines

Quick Review

Parallel lines have the same slopes.

The product of the slopes of two perpendicular lines is -1.

Example

What is an equation of the line perpendicular to $y = 2x - 5$ that contains $(1, -3)$?

Step 1 Identify the slope of $y = 2x - 5$.

The slope of the given line is 2.

Step 2 Find the slope of a line perpendicular to $y = 2x - 5$.

The slope is $-\frac{1}{2}$, because $2\left(-\frac{1}{2}\right) = -1$.

Step 3 Use point-slope form to write $y + 3 = -\frac{1}{2}(x - 1)$.

Exercises

Determine whether $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are *parallel*, *perpendicular*, or *neither*.

41. $A(-1, -4), B(2, 11), C(1, 1), D(4, 10)$

42. $A(2, 8), B(-1, -2), C(3, 7), D(0, -3)$

43. $A(-3, 3), B(0, 2), C(1, 3), D(-2, -6)$

44. $A(-1, 3), B(4, 8), C(-6, 0), D(2, 8)$

45. Write an equation of the line parallel to $y = 8x - 1$ that contains $(-6, 2)$.

46. Write an equation of the line perpendicular to $y = \frac{1}{6}x + 4$ that contains $(3, -3)$.